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Reinsuring Longevity: How Age Group Dynamics Influence Longevity Insurance Returns

Thesis

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Az elmúlt évtizedek rámutattak arra, hogy a hosszú és tartalmas élet többé már nem csupán álom, hanem valóság. Számos ipai és társadalmi tényező kedvező együtt állása hozzájárult az életkilátások példázatlan javulásához, melynek köszönhetően a várható élettartam is eddig nem látott szinteket ért el. A halandóság javulásával egyidőben az ebből vonatkoztatható kockázatok és kihívások is napvilágot láttak. Ezen kockázatok a hétköznapi szemlélő számára könnyen rejtve maradhatnak a pozitívumok takarásában, ugyanakkor jelentőségük nem tekinthető csekélynek. Jelen dolgozat célja, hogy a „hosszú élet kockázat" kérdéskörét, mint a halandóság javulásából származtatható egyik kihívást taglalja.

A hosszú élet kockázat teljeskörű megértése végett, a dolgozat kiemelt hangsúlyt helyez a jelenség hátterének, illetve lehetséges kockázatkezelési módozatainak bemutatására. Mindezt elsősorban a biztosítási iparág szempontjából téve, tekintettel arra, hogy ez az iparág van a legnagyobb mértékben kitéve a vizsgált kockázatnak. A dolgozat kutatási kérdéskörét tekintve, egy specifikus kockázatkezelési módozatra, a „Hosszú Élet Viszontbiztosításra" fókuszál. A központi kérdés, hogy hogyan is alakul a hosszú élet viszontbiztosítás megtérülése a viszontbiztosító szemszögéből különböző korcsoportokat tartalmazó viszontbiztosításba adott portfóliók esetén. Egészen pontosan, hogyha a viszontbiztosításba adott portfólió 60-70, 70-80, illetve 80-90 éves személyek szerződéseit tartalmazzák.

A dolgozat során végzett kutatás rámutatott arra, hogy a hosszúélet viszontbiztosítás megtérülése valóban függ a viszontbiztosításba adott portfólió életkor kompozíciójától. Továbbá a megtérülések eloszlása nem csak különbözött, de nagyobb szórás volt megfigyelhető a magasabb korcsoportú portfóliók esetében. Ebből következtetés képpen levonható, hogy a viszontbiztosítási kockázat magasabb idősebb személyek szerződéseit tartalmazó portfóliók esetében. Ez a jelenség konzisztens maradt azokban az esetekben is, hogyha a viszontbiztosítás árazása során 1%, illetve 2% árrést alkalmazott a viszontbiztosító. Fontos kiemelni, hogy a vizsgálatok Magyarországi személyeket tartalmazó portfóliók feltételezése mellett zajlottak.

Abstract

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Title: Reinsuring Longevity: How Age Group Dynamics Influence Longevity Insurance Returns

Keywords: Longevity risk management, Longevity Insurance

Living a long and prosperous life is a common dream, spanning our society from poor to rich. Owing to the various factors, this dream has become a reality over the past decades, with mortality improvements reaching unprecedented levels in human history. However, this remarkable rise in human life expectancy has also revealed adverse risks which may seem less obvious to the ordinary observers. The objective of the current research, among other purposes, is to examine a specific one of these emerging risks called "longevity risk", affecting mostly the insurance industry.

To gain better understanding of longevity risk, a comprehensive overview of longevity risk management solutions was conducted. Considering the main question of the research, it concentrated on a specific solution called Longevity Reinsurance or in other words, Insurance-Based Longevity Swap. The aim of the present research was to explore how the return of reinsurance contract varies when the demographic characteristics of the underlying reinsured population changes. Specifically, what is the impact of demographic factors, such as age groups, on the return of Longevity Insurance from the perspective of the reinsurer?

The research revealed that the return distribution of the reinsurance contract was significantly dependent on the age composition of the reinsured portfolio. Furthermore, the return distributions were not only different, but the standard deviation of the return increased for underlying reference portfolios consisting of older individuals. In other words, there is greater risk associated with reinsuring portfolios that include older individuals. This effect remained consistent in cases when additional margins were applied to ensure the profitability of the reinsurer.

The compared portfolios consisted of individuals aged 60-70, 70-80 and 80-90 while the applied margin cases were 0%, 1% and 2%. The nationality of the individuals in each portfolio was assumed to be Hungarian with the corresponding mortality characteristics.

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1 Introduction

Living a long and prosperous life is a common dream, spanning our society from poor to rich. Owing to the various factors, this dream has become a reality over the past decades, with mortality improvements reaching unprecedented levels in human history (Burger et al. 2012). However, this remarkable rise in human life expectancy has also revealed adverse risks which may seem less obvious to the ordinary observers. The objective of the current research, among other purposes, is to examine a specific one of these emerging risks called “longevity risk”, affecting mostly the insurance industry. The following chapters provide a detailed overview of longevity risk, initiating with a comprehensive introduction to mortality improvements and concluding with potential risk management solutions.

Longevity risk and its adverse effects impact insurance entities through the individuals they provide service to. Therefore, insurance entities frequently seek to mitigate this emerging risk, connecting to the increase in human life expectancy, by employing various risk management solutions.

While the research direction presented numerous possibilities, the central focus of the study revolves around a specific risk management solution called Longevity Reinsurance or in other words, Insurance-Based Longevity Swap. By creating a Longevity Reinsurance contract via simulation, the aim of the current research is to observe how the underlying portfolio’s demographic features, such as the age of the individuals influence the performance of the longevity risk management instruments. Specifically, whether the age of the individuals in the portfolio has significant impact on the return of the Longevity Reinsurance contract from the perspective of the reinsurer. To address this question, three underlying portfolios were constructed, “given into reinsurance” through Longevity Insurance and then compared. The underlying portfolios consisted of people aged 60-70, 70-80, and 80-90.

Considering the structure of the research, chapter 2 examines the theoretical background of mortality improvements and longevity risk management. Following this, chapter 3 outlines the research question and hypotheses, while chapter 4 presents the methodology employed. Finally, chapter 5 delves into the results obtained.

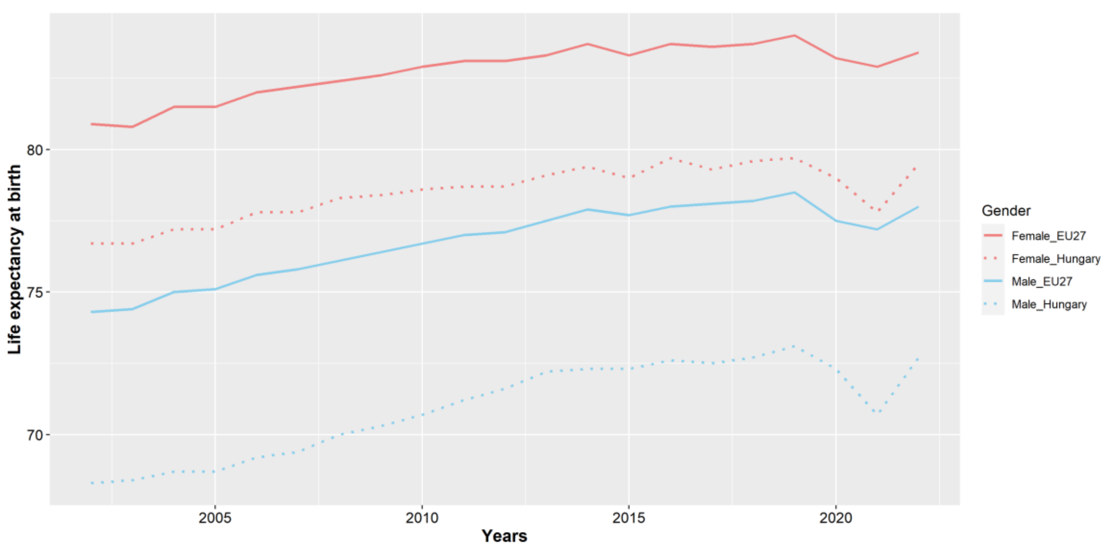
2 Theoretical background

2.1 Mortality improvements

One of the most significant advancements of our time is the remarkable rise in human life expectancy. Over the past century, mortality has undergone a rather accelerated decline, unprecedented in the known human history. The uniqueness of the development lies not just in its progressive nature but also in its equal impact across all age groups (Burger et al. 2012). Although the pace of the progression moderates (Raleigh, 2019), and its volume even tends to change in time (Vékás, 2020), the final boundary of the development remains unclear as it is expected to decrease even further during the upcoming decades (Ebeling, 2018).

Considering the personal benefits of mortality improvements, the warm welcome of the phenomenon is beyond dispute (Alburto et al., 2020). As entire nations live longer thanks to contributing factors such as medical improvements and food supply related advancements (Burger et al. 2012; Yue, 2012), human life expectancy has a seemingly clear path to rise unchallenged (Ebeling, 2018). Despite a temporary setback caused by the recent COVID-19 pandemic, mortality trends remain uninterrupted as illustrated in **Figure 1**.

Figure 1, Comparison of life expectancy developments by gender and country



Source: Own work, based on Eurostat (2024), Life expectancy by age and sex dataset.

Besides the rising trend of life expectancy, **Figure 1** sheds light on the significant difference between males and females. The female population not only has a better prospect for a longer life than their male counterparts when considering Hungary, but this identified difference also remains consistent when considering the European Union in general. Additionally, **Figure 1** also foreshadows **Table 1**, which presents twenty years of life expectancy development for 10 countries.

Table 1, Life expectancy by country

Year	Czechia	France	Hungary	Netherlands	Austria	Poland	Romania	Slovakia	Sweden	Norway
2022	79.1	82.3	76.2	81.7	81.1	77.4	75.3	77.2	83.1	82.6
2021	77.2	82.4	74.3	81.4	81.3	75.5	72.8	74.6	83.1	83.2
2020	78.2	82.3	75.7	81.4	81.3	76.5	74.2	77	82.4	83.3
2019	79.3	83	76.5	82.2	82	78	75.6	77.8	83.2	83
2018	79.1	82.8	76.2	81.9	81.8	77.7	75.3	77.4	82.6	82.8
2017	79.1	82.7	76	81.8	81.7	77.8	75.2	77.3	82.5	82.7
2016	79.1	82.7	76.2	81.7	81.8	78	75.2	77.3	82.4	82.5
2015	78.7	82.4	75.7	81.6	81.3	77.5	74.9	76.7	82.2	82.4
2014	78.9	82.9	76	81.8	81.6	77.8	75	77	82.3	82.2
2013	78.3	82.4	75.8	81.4	81.3	77.1	75.1	76.6	82	81.8
2012	78.1	82.1	75.3	81.2	81.1	76.9	74.4	76.3	81.8	81.5
2011	78	82.3	75.1	81.3	81.1	76.8	74.4	76.1	81.9	81.4
2010	77.7	81.8	74.7	81	80.7	76.4	73.7	75.6	81.6	81.2
2009	77.4	81.5	74.4	80.9	80.5	75.9	73.7	75.3	81.5	81
2008	77.3	81.4	74.2	80.5	80.6	75.6	73.5	74.9	81.3	80.8
2007	77	81.3	73.6	80.4	80.3	75.4	73.1	74.6	81.1	80.6
2006	76.7	80.9	73.5	80	80.1	75.3	72.5	74.5	81	80.6
2005	76.1	80.3	73	79.6	79.5	75	71.9	74.1	80.7	80.3
2004	75.9	80.3	73	79.3	79.3	74.9	71.4	74.2	80.7	80.1
2003	75.3	79.3	72.6	78.7	78.8	74.7	71	73.8	80.3	79.6
2002	75.4	79.4	72.6	78.5	78.9	74.5	70.9	73.8	80	79

Source: Own work, based on Eurostat (2024), Life expectancy by age and sex dataset.

While mortality improvements are undeniable in each observed country, its pace and magnitude differ significantly from nation to nation. For instance, Eastern European countries such as Hungary have both a lower initial life expectancy at the start of the observation period and do not reach the same level of life expectancy either, compared to their Western European counterparts. Based on these findings, it can be safely concluded that mortality improvements are consistent but are also highly influenced by the unique characteristics of individual nations, which viewpoint is also supported by Burger et al. (2012).

2.1.1 Longevity risk

The adverse consequences of mortality improvements emphasized in chapter 2.1 are commonly called „longevity risk”. Longevity risk generally refers to the exposure resulting from the uncertainty surrounding the development of aggregate mortality rates (Blake et al., 2006, Blake et al., 2019). In the present thesis, the concept of longevity risk is defined as the difference between the Net Present Value of Cash Flows (PVCF) under shocked mortality and the Net Present Value of Cash Flows (PVCF) under the best estimate mortality.

To gain a fundamental understanding about the risks posed by longevity, it is essential to define its individual and aggregate aspects. Simply put, individual-level longevity risk refers to the possibility that individuals may outlive their wealth due to longer-than-anticipated survival duration, which was unforeseen during their years of financial saving. On the other hand, aggregate-level longevity risk is considered in context of entire birth cohorts rather than individual people. If the cohort represents an aggregate of individuals born in the same year, then aggregate-level longevity risk refers to the possibility that the average member of the cohort experiences a longer-than-anticipated lifespan, leading to higher-than-expected average years of survival (MacMinn et. al., 2006; Stallard, 2006).

Regarding the nature of the longevity risk, it is not a surprise that the issue appears abstract to the ordinary observer, as it is mostly concerns them indirectly. Indeed, those entities most affected by the adverse implications of the longevity phenomenon are the ones with specific financial exposure to this domain. Specific financial exposure in the present context means services that operate over several decades and are highly reliant on the development of both individual and cohort mortality. Entities exhibiting such characteristics primarily include defined benefit (DB) pension funds and life annuity service providers as their operational stability is significantly threatened by mortality rate decrease that exceeds their preliminary projections, leading to substantial differences compared to their initial pricing and reserving calculations. In contrast, life insurance providers often benefit from improvements in mortality rates, underscoring the notion that what poses a threat to one service line may present an opportunity for another (Blake et al., 2006; Denuit et al., 2007; Blake et al., 2019).

Although the increase in human life expectancy had been acknowledged and observed for several decades, the potential negative consequences of longevity risk only gained widespread attention in December 2000. This notorious date marks the collapse of the Equitable Life Assurance Society (ELAS), the world's oldest life office at the time, an event that shook the previously well-founded confidence in the predictability of aggregate longevity. Incidents like this or the pension crisis encouraged both the regulatory authorities and entities exposed to longevity risk, to pay greater attention to the phenomenon and to seek out alternative financial solutions to mitigate their increasing exposures (Blake et al., 2008; Blake et al., 2019).

2.1.2 Affected entities

As the topic of mortality improvements has been thoroughly introduced in chapters 2.1 and 2.1.1, it is also essential not to overlook the industry that has emerged to address financial aspects of supporting elderly individuals. The affected entities share the common feature of providing lifelong services which traditionally means annuity products (Ngai & Sherris, 2011). By definition, annuities are financial products, which grant the purchaser a regular income. The frequency of the payments is typically monthly or annual, while the amount is determined by the size of the paid premium for which the annuity right was purchased (Blake, 1999). In general, annuities have many variations, and not all of them necessarily provide benefits until the death of the purchaser. To be precise, temporary annuities paid until death of the purchaser or until the end of the predefined contract term, whichever occurs sooner (Banyár, 2021). Even though all types of annuities are affected by mortality improvements, whole life annuities are the ones particularly influenced by the recent life expectancy trends (Ngai & Sherris, 2011).

Placing the annuities into a narrower context, these kinds of financial products are typically provided by entities of the insurance industry. In particular, life insurance companies and pension funds possess both the business expertise and legal foundation to engage into this specific market segment (Blake et al., 2019). However, according to Blake (1999), participants of the annuity market may not necessarily benefit from the rapid development of mortality. In fact, mortality improvement indirectly causes the extension of the payout period of annuity products (Tsai et al., 2011). As mortality rates fall faster than initially anticipated, the originally calculated purchase price becomes less and less sufficient. The issue becomes critical when the improving mortality rates diverge significantly from the rates used during pricing. This can render the product financially unstable, threatening both the promised payments and the insurance company itself

(Blake et al., 2006). For instance, mortality improvements are often miscalculated to the level of 20 percent difference within just 10 years (Blake, 1999).

In addition to pension funds and life insurance companies, national pension schemes are also experiencing the impact of the increased life expectancy prospects. Because of their implicit nature, Defined Benefit (DB) pension schemes are especially sensitive to mortality improvements (Blake et al., 2019). Based on Bodie et al. (1988) description, DB pension plans are characterized by their dependence on a specific calculation formula. This formula serves as the basis for the pension amount determination and usually takes into account the individual's years of work and size of their salary. In contrast, the other major type of pension systems relies on the consistent contribution of its participants. Defined Contribution Plans (DC) are built on the approach that employees or in some cases the employer makes regular in-payment to the individual's retirement account. The size of the in-payments is usually predetermined and calculated as the fraction of the employee's salary. As the years pass, the deposited amount increases through the repeated contributions. The assessment of the DC plan in any given time is the current market valuation of the assets within the retirement account.

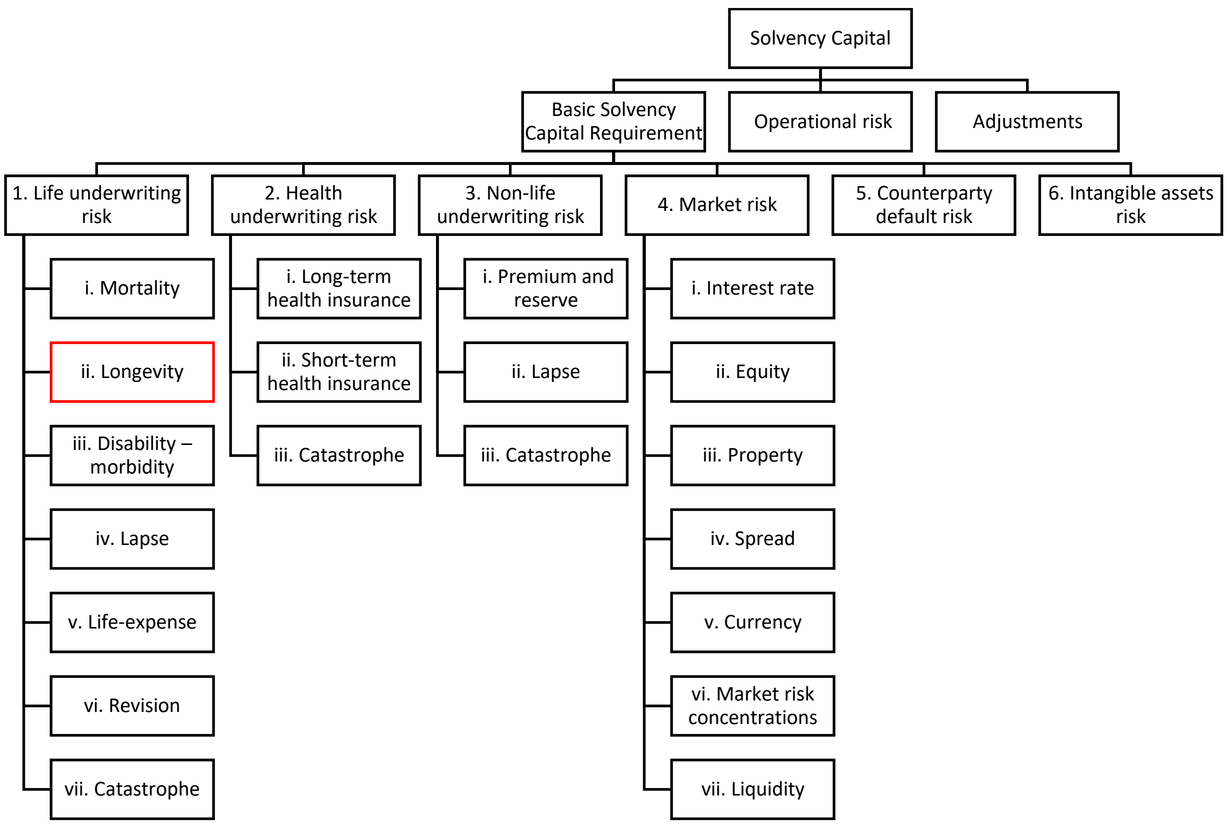
Basically, the benefit of both DB and DC pension plans can be considered as a lifetime annuity (Antolin, 2007). However, in the case of DC plans, the amount is determined by the accumulated yearly contributions, unlike DB plans, which rely on a formula without a supporting savings account (Bodie et al.,1988). Therefore, DB plans are more vulnerable to mortality improvements as the increasing life expectancy results in an extended payment duration that may not have been accounted for in the initial calculation of the formula. This reason besides other aspects such as decline in long term interest rates and increasing regulatory burdens effectively moves the personal and national preferences closer to DC based solutions. Although a historical trend of transitioning from DB to DC can be clearly observed, the process has experienced a significant acceleration over the past several decades and is expected to gain even more momentum in the near future (Broadbent et al., 2006).

2.1.3 Longevity risk and Solvency II

The present chapter serves as a brief overview of a special consequence stemming from longevity risk. Considering the novel directive of solvency capital requirements, entered in force on January 2016, for insurance companies and pension funds under the jurisdiction of the

European Union, the risk resulting from longevity got shed light on from a new perspective. Under the regime of Solvency II, insurance and reinsurance undertakings are obligated to define and keep a sufficient level of capital to cover the risks they face. To achieve this goal, companies are required to assess and manage their exposures comprehensively, including the risks arising from the improvement of mortality. In other words, insurance companies and pension funds should prove that the reserves they keep and the capital they possess are adequate and are able to cover potential losses emerging from, for instance, longevity (EU, 2009). The elements of the risk management framework proposed by the Solvency II directive is illustrated in **Figure 2**.

Figure 2, Components of solvency capital requirements under the regime of Solvency II



Source: Own work, based on Vékás (2016, p. 19)

The Basic Solvency Capital Requirement is determined based on individual risk modules (represented with Arabic numbers) which are further divided into submodules (lowercase roman numbers). The calculations are done on submodule basis first, and then aggregated into the risk modules. Besides „Basic Solvency Capital Requirement", the "capital requirement for operational risk" and the "adjustment for the loss-absorbing capacity of technical provisions and deferred taxes" serve as the basic for the calculation formula of Solvency Capital Requirement. Longevity risk (ii), as the focus of the present paper, represents one of the submodules of the life underwriting risk (1.) module (EU, 2009).

Solvency Capital Requirement (SCR) is defined to ensure that insurance companies are able to meet their payment obligations with the probability of 99,5% over a one-year period. To rephrase, by meeting the Solvency Capital Requirements, insurance companies are not expected to become financially distressed due to adverse events more than once in every two hundred years. In order to accomplish this objective, the risks identified in the submodules are aggregated to get the SCR for individual modules and the same aggregation applies to the modules to get the SCR for the Basic Solvency Capital Requirement. This process involves taking into account correlations at both the submodule and module levels (EU, 2009; Vékás, 2016).

The Solvency II regime and therefore longevity risk itself have direct impact on the amount of capital held by insurance and reinsurance undertakings (EU, 2009). For a company, regardless of its industry, keeping capital incurs emerging costs that can be characterized from several aspects. One way to properly assess it is through financial assets like bonds (Modigliani & Miller, 1958). Due to its inherent nature, longevity exposure is considered a particularly capital-intensive risk within the current framework. Longevity risk is typically regarded as a trend risk with a long-term perspective. Given its low probability to experience significant fluctuations within a short period, the capital reserved for extreme longevity scenarios is only assumed to be required over decades. Hence, insurance and reinsurance undertakings are strictly bound by regulations to limit the transferrable longevity-related liabilities as the associated risks primarily have remote consequences (Michaelson & Mulholland, 2014).

Considering the insurance industry, cost-of-capital clearly reflects in product pricing, particularly regarding life annuities. Due to the costs associated with holding capital, insurance companies have a great interest in finding alternative solutions to reduce their longevity exposure. By doing so, they can decrease the amount of funds they are required to hold as solvency capital, resulting in cost-saving benefits. Longevity versions of financial instruments, such as longevity swaps, offer a viable solution as these instruments are capable of reducing solvency capital requirement by mitigating longevity risk (Meyricke & Sherri, 2014). Financial instruments exhibiting these capabilities are presented in more detail in subsequent chapters.

2.2 Longevity risk management

Before delving into the introduction of financial instruments devised to mitigate substantial damages resulting from longevity risk, it is worth devoting a thought of the potential size of the global longevity risk market. To capture the sense of grandiosity, Michaelson and Mulholland (2014) quantified the global longevity risk market through the aggregate accrued liabilities of the developed world's retirement systems. In total, the accumulated retirement obligations of the world's developed economies were estimated to fall in the astonishing range of $60 trillion to $80 trillion already in 2012! This defined range is a subject to the development of mortality rate improvements. Each year with an unexpected rise in the average lifespan of individuals who have already reached the age of 65 implies a 4–5% increase in the global pension liabilities (Swiss Re Europe, 2012). In the current context, an unanticipated rise in the average lifespan means an increase of mortality improvements by 0.8%, or alternatively, a decrease in mortality rates by 13%. Based on approach of the Risk Management Solutions (2014), it is possible to calculate the standard deviation for a sustained shock of annual mortality improvements. Considering a shock of mortality improvements lasting more than 10 years, the standard deviation is estimated to be approximately 0.80% when compared to expected level. Michaelson and Mulholland (2014) utilized this calculation to determine the effect of a longevity tail event. To be precise, a 2.5 standard deviation event coincides with a trend change of 2% ($0.80\% \times 2.5 = 2\%$) resulting in an increase in longevity-related liabilities of about 10–12.5%. Therefore, an unforeseen rise in life expectancy may lead to the escalation of global retirement obligations by an additional $5–8 trillion or even more.

There have been several techniques developed throughout the past decades to mitigate the financial exposure, emerging due to the risk associated with longevity. In professional terms, the practice of mitigating financial exposure is commonly referred to as 'hedging'. Hedging is a financial strategy aimed at reducing the risk of unfavourable price movements in assets. It involves taking proactive measures to reduce the variability of cash flows, thereby minimizing the probability of incurring significant losses or bankruptcy costs (Kim et al., 2006). In the following sections the two mainstream directions of longevity risk hedging will be presented: Insurance Based Solutions and Capital Market Solutions.

2.2.1 Insurance-Based Solutions

Insurance-based hedging solutions are widely considered the traditional way of dealing with undesirable longevity risk. The term includes three main methods: Pension Buy-outs, Pension Buy-ins and Insurance-Based Longevity Swaps, the latter also known as Longevity Reinsurance (Blake et al., 2019). Considering the approach of Michaelson and Mulholland (2014, p.20), the listed hedging alternatives often referred to as “pension risk transfer contracts”, besides the used naming conventions. The naming essentially depicts the underlying functions as the purpose of these methodologies is to facilitate the transfer of longevity risk from public and private pension funds to a wider range of risk takers. Since many actors at longevity risk often lack sufficient compensation and appropriate risk management tools, they have become increasingly interested in opportunities to transfer the affected liabilities off their balance sheets using the aforementioned methods. One of the most publicized instances include the case of General Motors (GM) and Prudential Financial in November 2012. During the transaction, General Motors transferred $29 billion in pension plan assets. In exchange, Prudential took on the responsibility of paying the $26 billion pension owed to approximately 110,000 retired GM employees in the United States. This exchange allowed GM to reduce economic volatility associated with the financing of the pension plan and to improve its valuation transparency at the same time (Morgan Stanley, 2012).

While insurance-based hedging solutions share the same purpose, their way of achieving the desired outcome differ in several aspects. The main differences are manifested in the management of assets in question, the bearing of investment risk, and the administration of pension payments. Particularly regarding the allocation of responsibilities among the actors, participating in the transaction. After highlighting their differences, it is important to also emphasize one of their main common characteristics, in addition to their same goal. Insurance-based hedging solutions offer a high level of customization due to their nature, leading to full compensation for the specific risks faced by the hedger (Michaelson & Mulholland, 2014). Because of this feature, hedging solutions of this kind are classified as “customised indemnification solutions” (Blake et al., 2019, p. 8).

The clientele of Insurance-Based hedging solutions is highly dependent on the counterparty. Although investment banks are often associated with Longevity Swaps, Insurers and DB pension plans constitute the main target audience of the mentioned solutions. Therefore, in the following subsections the entity buying the hedge will often be referred to as „pension plan." (Kiff, 2022).

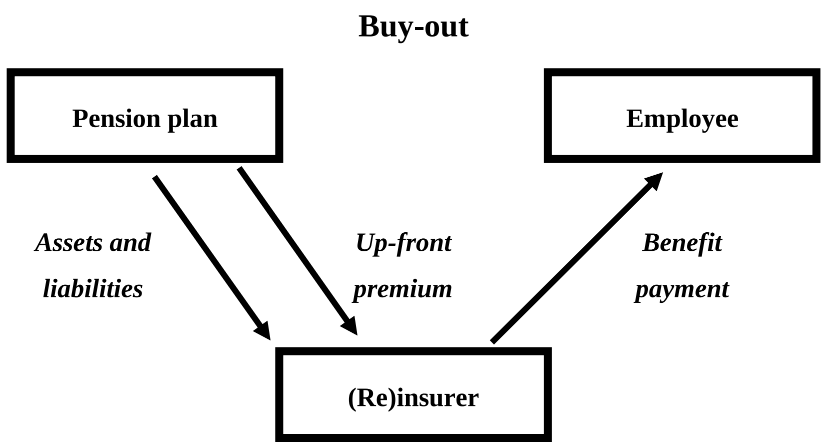
2.2.1.1 Pension Buy- outs

Pension Buy-outs represent the oldest traditional solution for addressing longevity exposure. During a buy-out transaction, the hedger transfers all its liabilities at risk, supplemented with an initial up-front premium. The up-front premium serves as the compensation to the other party in the transaction for taking over the pension obligations (Kiff, 2022).

As the first extensively introduced insurance-based hedging alternative, the detailed example of Blake et al. (2019, p. 9) is presented to be able to grasp the essence of the method. Consider two companies: a pension fund called ABC and a life insurer company XYZ. The pension plan assets (A) of ABC are valued at 85, while the pension plan liabilities (L) are valued at 100, resulting in a deficit of 15. Assuming that ABC approaches XYZ with the opportunity of a full pension but-out, XYZ values the pension liabilities of ABC at 120. This means that compared to the valuation of ABC company's actuary, there is an additional premium of 20 increasing the deficit from 15 to 35. Due to diligence, XYZ also takes on the assets of ABC. Therefore, ABC has the obligation to additionally contribute 35 (120-85=35) from its internal or external resources (like borrowing).

Although the former example may not seem a good deal for the first sight, ABC reaches its initial objective: hedging its longevity exposure. Suppose ABC lacks the sufficient resources and decides to take on, for example, a loan. In this case the financial exposure associated with the loan - such as interest rate and inflation risks - is less volatile and better understood by investment analysts and shareholders than the risk associated with the fluctuation of pension liabilities. Nevertheless, regardless of the sourcing of the additional contribution (35), ABC eliminates the volatility in its profit and loss (P&L) accounting originating from the pension plan by completely removing balance sheet liabilities associated with pension obligations. On the other hand, potential disadvantages of the transaction may emerge because of its timing. Buy-outs are final agreements that cannot be modified, even if the originally determined exchange price – such as the valuation of 120 in the previous example – turns out to be miscalculated due to the change of future circumstances. For instance, the value of pension liabilities may increase beyond previous expectations due to the rise in long-term interest rates and the resulting change in discount rates used for the original valuation (Blake et al., 2019). The structure of pension buy-outs is presented in **Figure 3**.

Figure 3, Structure of pension buy-out transactions

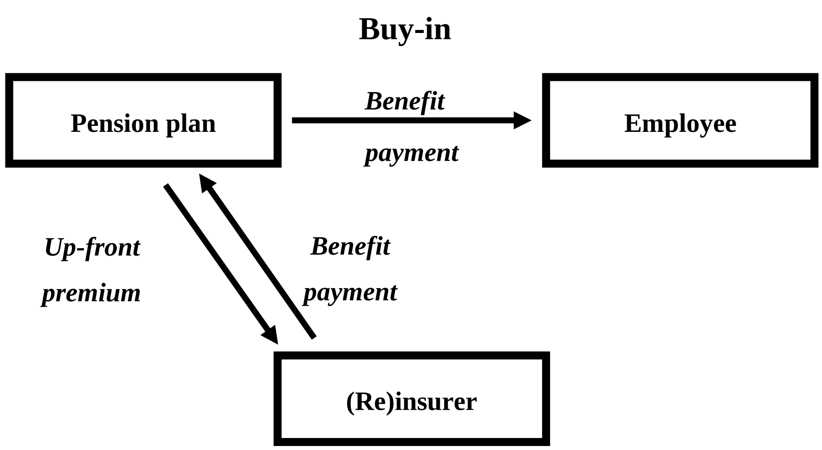


Source: Kiff (2022, p. 209)

2.2.1.2 Pension Buy- ins

In contrast to Pension Buy-outs, Pension Buy-ins utilize a different approach to neutralize exposures arising from longevity risk. In case of Buy-ins, neither the assets nor the liabilities of the pension plan change hands. Instead, the pension plan purchases financial instruments which provide periodic payments, in sufficient amount to be able to cover its arising pension obligations. Financial instruments with such properties include annuities for instance. Similarly to Buy-outs, the pension plan pays compensation for the annuity provider in a form of initial up-front premium (Kiff, 2022). The purpose of the purchased annuities is to serve as risk cover for specific mortality characteristics, associated with a part of the pension plan's liabilities. Although these mortality characteristics are defined by the plan's beneficiaries – such as their age, gender, and paid pension amount – there are no annuity certificates issued for the individuals. Therefore, the purchased annuities do not become assets of the individual members but rather assets of the pension plan itself. Because there is no transaction in place which affects the pension plan liabilities, those are not removed from the pension plan's balance sheet either. This stands in contrast to Buy-outs, where balance sheet liabilities associated with pension obligations are completely removed. Despite this difference, buy-ins can be considered as a step towards a full buy-out. Buy-ins inherently have a de-risking property from economic point of view which can be leveraged if the bulk purchase of annuities happens in phases. By this the pension plan can stabilize annuity rates over time and prevent a sudden increase in pricing when transitioning to a full buy-out. Additionally, buy-ins provide the sponsor with the benefit of fully immunizing a portion of the pension liabilities for a reduced initial cash payment compared to a full buy-out (Blake et al., 2019). The structure of pension buy-in transactions is presented in **Figure 4**.

Figure 4, Structure of pension buy-in transactions

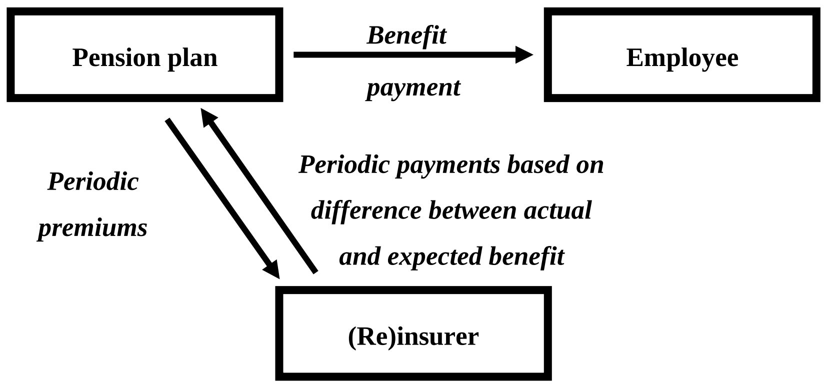


Source: Kiff (2022, p. 209)

2.2.1.3 Longevity Reinsurance (Insurance-Based Longevity Swaps)

In terms of transaction structure, Longevity Reinsurance demonstrates considerable similarities to Pension Buy-ins. As **Figure 5** represents, one of the main differences manifests in the nature of transactions between the reinsurance buyer (Pension plan) and reinsurance provider (Reinsurer). In case of Longevity Reinsurance, the up-front premium is replaced by periodic premiums distributed throughout the duration of the contract (Kiff, 2022). The periodic premiums are pre-fixed payments determined at the beginning of the contract, paid by the reinsurance buyer. The basis for the calculation of the pre-fixed payments is the expected benefit payment for the pension portfolio which reflects the expected longevity risk at the time of signing the contract. In exchange, the reinsurance buyer receives payments based on the realised mortality experience of the portfolio participating in the contract. The received benefit payments can then be used by the reinsurance buyer to cover its arising pension obligations (Blake et al., 2019).

Figure 5, Structure of a Longevity Reinsurance contract



Source: Kiff (2022, p. 209)

During a Longevity Reinsurance, no asset transfers take place, which allows the pension plan trustees to maintain control over the asset portfolio. Therefore, the pension plan is able to remove its longevity exposure via a highly customized, long-maturity hedging solution (Blake et al., 2019).

As Blake et al. (2019) mention it in their comprehensive work, the first widely known Longevity Reinsurance transaction took place in April 2007 and involved Swiss Re and UK life insurer Friends' Provident. The transaction was a pure longevity risk transfer and was based on Provident's £1.7bn book of 78,000 pension annuity contracts. By the swap, Swiss Re took on the longevity risk in return for an undisclosed premium.

2.2.2 Longevity risk management - Capital Markets Solutions

The longevity risk management tools, introduced in the previous chapters, all belonged to the group of Insurance-Based solution and shared the common feature of high customization. In other words, these solutions offered full compensation to the hedger as they could be tailored to cover their specific hedging needs (Michaelson & Mulholland, 2014). Unlike Insurance-Based solutions, Capital Market Solution cannot be customized because the participating portfolio is not the hedger's own, but rather an independent one to which the solution is linked (Zhou & Li, 2017). In most of the cases, the independent portfolio is basically a cohort, for instance the English and Welsh males aged 65 at the time of the contract. The payments of the capital market instruments are connected to the underlying cohort, and the survivor rate of the members forming it. The main types of capital market-based longevity hedging instruments are longevity bonds, longevity swaps, q-forwards and longevity options. Given that underlying cohorts exhibit both mortality and survivor rates, there also exist inverse versions of the mentioned instruments which are linked to mortality directly (Blake et al., 2019). Moreover, these longevity- and mortality-linked securities have standard features, which would be typically expected of regular bonds, swaps, forwards, and options (Blake et al., 2006).

Since the survivor rates used for the development of hedging instruments are widely accessible, these instruments possess several features that make them more appealing to a broader audience. Capital market participants may also express interest in the longevity asset class because of its minimal correlation with other asset classes, providing great diversification opportunities. Besides, the standardized nature of longevity assets promotes the development of liquidity while removing potential information asymmetry. Information asymmetry is a typical concern of Insurance-Based Hedging solutions as pension plans fundamentally possess better knowledge about mortality outcomes within their own portfolio (Zhou & Li, 2017).

Of course, there is also an important downside of the standardized nature of the capital market based hedging solutions. Given that standardized longevity assets do not offer a perfect solution, residual risk remains in every case. The most prominent part of all the residual risk components is basis risk, which reflects the difference arising from the mortality improvements of the underlying portfolio and the hedger's own portfolio (Zhou & Li, 2017). Before delving into the detailed introduction of capital market based hedging solutions, the addressing of basis risk will take place.

2.2.2.1 Concept of basis risk

Population basis risk is related to the fact that there is no capital market based hedging solution, which does not involve the possibility of potential mismatch between the populations of the underlying exposure and the hedge. This holds true for hedging instruments designed to address longevity and mortality risks as well. By definition, population basis risk arises as a hazard reflecting that the actual mortality/longevity outcomes of a population may easily differ from the characteristics connecting to the population defined by the hedging instrument

(Coughlan et al., 2007). For instance, such risk can originate from variations in socioeconomic status, lifestyle choices, and geographic location, among other factors (Li & Hardy, 2011).

Considering the point of view of Village et al. (2017), three aspects can be regarded as the root cause of basis risk. First of all, structuring risk of the payments occurs when, for example, the hedging instrument has an annual payment structure while the entity hedges a service which provides monthly payments. This scenario is easily imaginable for life annuity providers and pension plans. Additionally, according to their interpretation population basis risk can be subdivided into two components: sampling risk and demographic risk. Demographic risk is owed to the emerging differences between the hedged and actual portfolios due to demographic and socio-economic reasons. Therefore, this type of risk leads to divergent underlying rates both in the present and future. On the other hand, sampling risk stands the closest to what was defined as population basis risk by other authors. Sampling risk represents the disparity in mortality rates observed between the index and the actual portfolios, resulting from the inherent fluctuations regarding the individual lives within the portfolios.

Li and Hardy (2011) in their comprehensive work provide a thorough examination about models measuring and modelling basis risk. The authors also highlight relevant considerations, such as the insight formerly articulated by Coughlan et al. (2007), that the presence of basis risk does not necessarily indicate inefficiency during the hedging process. In fact, it is crucial to strive for the minimalization of basis risk. However high hedge efficiency is possible even in situations where basis risk is not negligible.

2.2.2.2 Longevity Bonds

Before delving deeper into the concept of longevity bond, it is essential to clarify the common understanding of the term 'bond'. However, it is important to highlight that the comprehensive introduction of bonds is out of the scope of the current work. The same applies to the other examined financial assets such as swaps, forwards, and options as the focus is on their versions related to longevity.

According to Thau (2001), bonds can be viewed simply as loans. Entities have the opportunity to express their need for additional capital by selling bonds, or in Wall Street terms, by issuing them. When purchasing a bond, the buyer agrees to lend money to the bond issuer in a legal framework which imposes payment obligation on the issuer. These payment obligations include the repayment of the original sum on a pre-agreed date and interest payments occurring periodically until the stipulated date. In official terms, the original sum called “principal”, the periodic interest payments called “coupons”, while the pre-agreed end date is referred to as “maturity date”. Bonds can be issued by various entities such as corporations and governments with various maturity dates ranging from a few days to 30 years. Following the example of the USA, „Treasuries” are bonds issued by the U.S. Government while “Municipals” are bonds issued by local and state governments. To bonds issued by corporations, the name “corporate bonds” is used (Thau, 2001).

Let's consider the example provided by Thau (2001) as a simple illustration of bonds. Assume that you invest $10.000 in a 30-year bond. If this bond has a semi-annual coupon payment of 7%, then the received amount every 6 months would be $350. In case of holding the bond until maturity, the total number of coupon payment would be 60, totalling $21,000 in payments. Further assuming a “bullet bond” the $10.000 principal is also repaid on the maturity date (Thau, 2001 p.52).

Another great example is provided by Menoncin (2008), illustrated in **Table 2**. Assume a longevity bond, with payments linked to an arbitrary reference cohort. Then the coupon payments of this bond would be based on the cumulative survival rate, calculated as the product of all previous survival rates (99% × 98.8% × 98.5% = 96.345). Therefore, the coupon payment in year 2007 would be $963.45 on a principal value of $1000 (Menoncin, 2008, p.345).

Table 2, Coupon structure of a longevity bond

Year	**2005**	**2006**	**2007**
Mortality rate (%)	1	1.2	1.5
Survival rate (%)	99	98.8	98.5
Cumulative survival rate (%)	99	97.812	96.345
Coupon (on £1000)	990	978.12	963.45

Source: Azzoppardi (2005) in Menoncin (2008, p.345)

In general, longevity bonds are similar to traditional bonds. For instance, consider the first classical longevity bond proposed by Blake and Burrows (2001). This longevity bond, also called „survivor bond” has its coupon payments linked to the survival rate of a given population cohort. If the reference cohort is a nation’s age group of 65 in 2002, then the coupon payment of this longevity bond in 2020 would be directly connected to the proportion of people alive in 2020 from the reference cohort. In other words, the coupon payment of the longevity bond would be based on the number of 85-year-olds in 2020 compared to the number of 65-year-olds in 2002.

Given that longevity bonds are effective instruments for mitigating aggregate mortality risk, it is not surprising that various versions have been developed over the past decades. Broadly speaking, longevity bonds can be classified into two main categories: „principal-at-risk" longevity bonds and "coupon based" longevity bonds. Regarding the former, the investor faces the direct risk of losing parts (or all) of the principal in the event of an adverse mortality event. On the other hand, coupon-based longevity bonds are characterized by the fact that their coupon payment is mortality dependent. The nature of this dependency can be different and defined by the kind of bond itself. Because coupon-based longevity bonds are designed for hedging purposes, it is reasonable that these kinds of bonds lack principal repayment at the end. Actually, most of these bonds have no predetermined maturity date either as their terms are connected to survival rates. From another point of view, these bonds often provide coupon payments until the last member of the cohort dies, whenever it occurs (Blake et al., 2006, p. 168).

The first real life transaction connected to longevity bonds occurred in 2002 and was specifically a pure mortality bond release. The issuer was the Swiss Re, who linked the principal payment of the bond to adverse mortality risk scenarios. Swiss Re issued the bond through a special purpose vehicle (SPV) called Vita Capital I, with an issue size of $400 million. Although the floating coupon payment was determined generously, the fact that the principal itself was in risk balanced the generosity. Precisely, the floating coupon rate was set as U.S. LIBOR plus an additional 135 basis points. In comparison, the principal payment was linked to a mortality index calculated as the aggregate average of mortality rates across five reference countries. The average was weight by considering the participating countries: United States, UK, France, Italy, and Switzerland (Dowd et al., 2006). The mortality index was referenced to the year 2002, and subsequent yearly indexes were compared to it until the bond matured in 2005. The principal payment was determined as follows: If the index did not exceed 1.3 times the 2002 base level in any of the years, then the principal would be repaid in full. Alternatively, if the index exceeded 1.3, the principal was reduced, and if it exceeded 1.5 times the base level, no repayment was made from the principal at all. In 2005 the bond reached maturity as planned, with no loss of principal to the investor (Chen & Cummnins, 2010). Bearing in mind the mortality improvements experienced at the beginning of the 21st century, the probability of high mortality was already deemed to be low. Therefore, the bond could be regarded as a higher-than-average coupon rate investment opportunity in return for potential exposure to some extreme mortality risk (Dowd et al., 2006).

A significant experiment in long-term longevity bonds was conducted through the instrument issued by the European Investment Bank (EIB) in 2004. BNP Paribas arranged the release of the bond, which amounted to £540 million and had a term of 25 years. The main target audience for this bond included pension plans and annuity providers, as its structure was similar to survivor bonds. (Chen & Cummnins, 2010). Investors of the bond were compensated with annual coupon payments of £50 million multiplied by a realized survivor index. The survivor index was based on data obtained from the UK Office for National Statistics regarding the population of English and Welsh males aged 65 in 2002. Nevertheless, despite the effort and collaboration of EIB (issuer), BNP Paribas (designer) and Partner Re (longevity reinsurer), the bond was not launched due to insufficient demand and was subsequently withdrawn in late 2005 (Chen & Cummnins, 2010).

2.2.2.3 Longevity Swaps

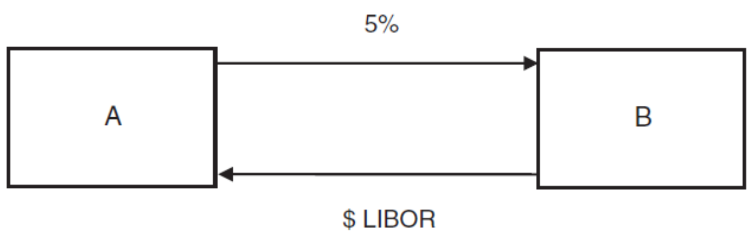
Swaps, alongside with forwards and options, belong to the category of derivatives. Derivatives are financial instruments whose value is derived from other assets, known as underlying assets. Within the family of derivatives, swaps are contracts between two parties in which they agree to exchange cash flows on regular dates. The cash flows involved in the agreement are referred to as payment legs which have different calculation basis. The calculation basis of the different legs is directly related to the underlying assets, which assets also determine the type of the swap contact. If the underlying assets are stocks or stock market indexes (like the S&P 500) on which the payment leg is based on, then we can talk about equity swaps. In other cases, if one leg is connected to a physical commodity price (e.g., oil), then the swap in question is a commodity swap; otherwise, if both legs are connected to interest rates, then the swap is an interest rate swap (IRS). Swaps are widely used derivatives among financial market participants as exposure management tools. Besides taking speculative trading positions, swaps are appropriate to hedge against the fluctuations of interest rates, currency exchange rates, stock prices, commodity prices, and loan defaults (Chisholm, 2010, p.59).

To help the proper understanding of general swaps before introducing longevity swaps, consider the example provided by Chisholm (2010, p. 61). Assume the most common version of an interest rate swap (IFS) with contracting parties A and B.

- **Fixed leg**: "A" company agrees to pay annually to "B" company a fixed rate of 5% on a notional amount of $100 million. This means a $5 million cash flow per year from "A" to "B".
- **Floating leg**: "B" company agrees to pay annually to "A" company the 12-month dollar LIBOR rate on the same notional amount, $100 million.

The exchange of the notional principal ($100 million) is not part of the three-year contract, starting at the moment of the agreement. LIBOR means the London Interbank Offered Rate, serving as a benchmark rate in global bank transactions. Because of the contract's three-year duration, there will be three payments, made annually. The illustration of the swap's payment structure is presented in **Figure 6**.

Figure 6 Legs of the interest rate swap



Source: Chisholm (2010, p. 62)

For considering the swap payment structure in the first year, let's assume first that the LIBOR rate is set at 4.5%. This means that "A" gives a $5 million to "B" and in return receives $4.5 million (4.5% of $100 million) as the floating leg rate. Therefore, in net, "B" receives $0.5 million from "A". If the LIBOR rate is reset at 5.25% in the second year, then the net payment direction changes and „A" receives $0.25 million ($5.25 million - $5 million) from „B" (Chisholm, 2010, p. 62).

Longevity and mortality swaps have similarities with traditional swaps. In both cases, the exchange of future cash flows depends on the outcome of at least one survivor index serving as leg. However, mortality swaps specifically hedge against the risk of mortality rates being higher than expected (Cox & Lin, 2007), while longevity swaps hedge against the risk of people living longer than expected (Blake et al., 2019). Therefore, despite their mutual aim of hedging longevity risk, the two instruments reach their goals by different means.

By definition, survivor swaps are formal agreements on exchanging future cash flows based upon the performance of at least one (typically random) survival index. Compared to survival bonds, survival swaps have considerably lower transaction costs, complemented with a more flexible and customizable structure (Dowd et al., 2006). Based on the analysis of Dowd et al., (2006, p.3) the mechanism of survival bonds can be illustrated the following way. In the most standard case, assume a mortality swap with a fixed leg involving a single present payment and a floating leg consisting of a single random payment depending on mortality development. In other words, the swap consists of an initially set amount of $K(t)$ and a random amount of $S(t)$, where „t" means a future time when the swap concludes. The two participating firms are "A" and "B", agreeing upon the exchange of only the net difference between the payment amounts. It means that "A" pays "B" the amount of $K(t) - S(t)$ if $K(t) > S(t)$, and "B" pays "A" the amount of $S(t) - K(t)$ if $K(t) < S(t)$. To be precise, the mortality dependent random amount ($S(t)$) is calculated based on the number of people alive from the initially specified, underlying reference portfolio. Such reference portfolio may consist of a portfolio of an annuity holder (Dowd et al., 2006, p. 4).

Cox and Lin (2007, p.9) described a more complex example of mortality swaps, where both legs are mortality-dependent, resulting in a swap of specific longevity risk for a different longevity risk. Assume an annuity provider and a life insurer. For the annuity provider, the adverse scenario is when the mortality rates decrease and, therefore, the people in its portfolio live longer than anticipated, resulting in additional payments. For the life insurer, the adverse scenario means the opposite: mortality rates increase and, therefore, more people die in its portfolio than it was anticipated. As the annuity provider and the life insurer have hedging intentions with opposite directions, it is logical if they enter into a mortality swap. In light of the example, the annuity provider pays a floating cash flow based on the realised mortality in the life insurer's portfolio and in return, the life insurer pays a floating cash flow based on the number of people surviving in the annuity provider's portfolio. Therefore, if mortality increases, the life insurer receives a net benefit payment from the annuity provider. This can be covered by the decreasing obligations of the annuity provider, as it has to pay annuities for fewer people due to the higher-than-anticipated death rates. Conversely, if mortality decreases, the annuity provider receives a net benefit, which is covered by the life insurer's funds. This is because the life insurer has fewer people to pay life insurance to, due to the lower-than-anticipated death rates (Cox & Lin, 2007, p.9).

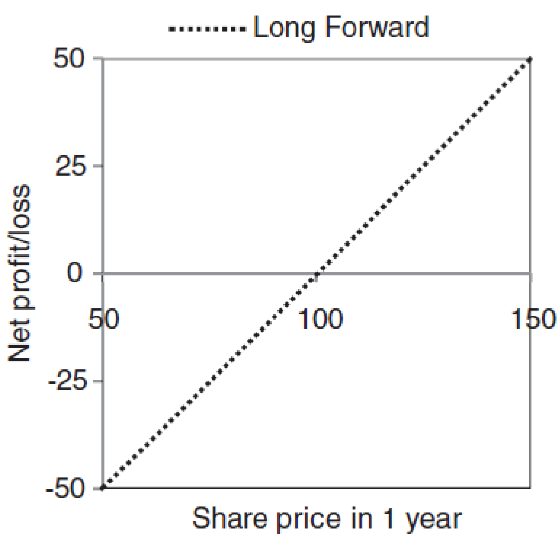
Within the context of longevity swaps, a real-world example is emphasized as follows. The first longevity swap occurred between J. P. Morgan and Canada Life in July 2008. The contact consisted of a £500 million longevity swap with a 40-year maturity. Regarding the underlying portfolio, it was characterised by 125,000-plus annuitants whose actual mortality experience determined the swap. Despite being customized rather than index-based, the transaction is still considered pioneering from a capital market perspective. Since the longevity risk was transferred from Canada Life to J. P. Morgan and subsequently to investors, it was the first of its kind to introduce capital market investors to the longevity market (Blake et al., 2019, p. 14).

2.2.2.4 q-Forwards

Forward contracts also belong to the category of derivatives, as their value is derived from underlying assets, for instance, commodities like oil or different financial assets such as shares. By definition, forward contracts are mutual agreements between two parties, with one party agreeing to sell and the other agreeing to buy the underlying assets. The transaction takes place on an initially fixed date on an initially fixed price. Another important feature of forward agreements is that no cash flow or transaction occurs on the date of the contract; rather, they are executed only on the initially specified future date. Forward agreements can be physically delivered, or cash settled. In the case of cash-settled contracts, only the difference between the prespecified fixed price and the actual market price of the underlying asset is paid on the predetermined future date. While forwards are typically tailor-made agreements, there exists a more standardized alternative as well. “Future” contracts are essentially the same as forward contracts with the difference that these are organized through regulated exchanges instead of direct negotiations between the two participating parties (Chisholm, 2010).

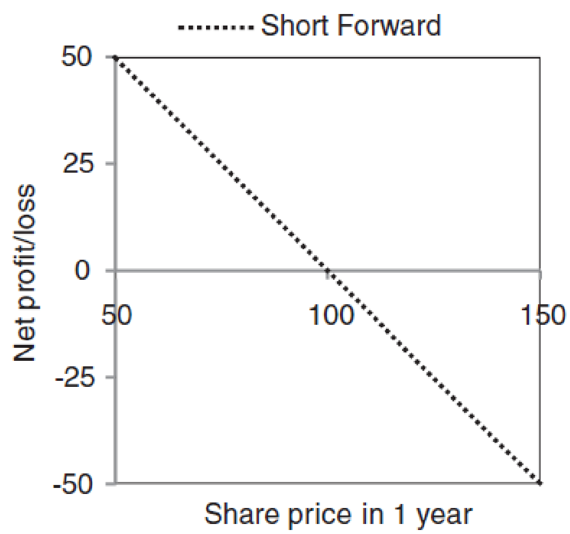
Consider the example of Chisholm (2010, p.19) to shed more light on the structure of forward agreements. Assume an equity forward contract where trader „A” agrees with trader „B” on purchasing a share exactly a year later at a fixed price of $100. This position, involving the purchase of the share is called a **long forward position.** Based on the possible share values at the point of delivery, there are multiple profit and loss (P&L) scenarios from the trader’s perspective, which are illustrated in **Figure 7**. For example, if the share is worth $150 in a year, then after the purchase for $100, the trader can sell it with a $50 profit. On the other hand, if the share is valued at $50 in a year, then after the mandatory purchase the trader can pass it on with $50 loss. From the perspective of the other trader, a **short forward position** was entered by agreeing upon the obligatory sell of the share on the prefixed date. Assume that the selling counterparty has no share in its possession and has to buy it on the prefixed date in order to sell it to the trader in the long forward position. In case the share has lower valuation than $100, then the selling trader will realize profit, while if the share costs more than $100, then the trader will realize loss. For instance, if the share costs $150 on the delivery date, then realized loss of the short position is $50 which equals with the realized gain of the long position. The P&L scenarios of the selling trader in the short position are illustrated in **Figure 8** (Chisholm, 2010, p. 17).

Figure 7, Profit and loss on long forward position



Source: Chisholm (2010, p. 18)

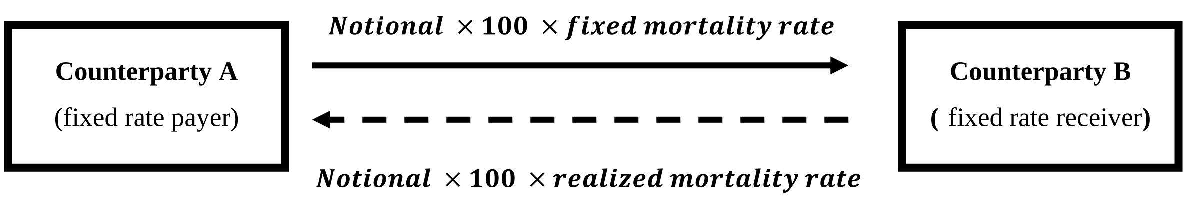
Figure 8, Profit and loss on short forward position



Source: Chisholm (2010, p. 18)

Longevity and mortality forwards represent the simplest type of derivatives of their kind, similarly to the traditional forward contracts. The name „q-forward" for mortality forward rate contracts originates form actuarial denotations. Precisely, the letter "q" symbolizes mortality rates both in actuarial practice and denotation. Essentially, a q-forward is a contract between two parties agreeing to exchange amounts proportional to mortality rates. One of the amounts is linked to the realized mortality rate of a given population, while the other amount is proportional to an initially fixed mortality rate. The exchange takes place on an initially set future date, called maturity date of the contract. Following professional terms, q-forwards can be also described as zero-coupon swaps, exchanging fixed mortality for realized mortality at the maturity date. This interpretation is illustrated in **Figure 9**, which utilizes the fact that realized mortality rates are usually linked to appropriate reference indexes such as the one published by LifeMetrics. Based on the definition of Coughlan et al. (2007b, p. 6), LifeMetrics is a toolkit designed for measuring and managing longevity and mortality risk, tailored for pension plans, their sponsors, insurers, reinsurers, and investors. **Figure 9** present the transaction on the maturity date of the contract.

Figure 9, Structure of q-forwards



Source: Coughlan et al. (2007, p. 2)

The importance of q-forwards is perfectly shown by their building block nature in the construction of more complex life-related derivatives. As Carins et al. (2008, p. 108-109) highlighted, a combination of q-forward contracts with various ages and maturities can be utilized to hedge a longevity swap. Suppose a longevity swap contract consisting of a fixed cash flow leg and a floating leg which has cash flow payment based on a realized survivor index.

Let's denote the fixed cash flow with $\hat{S}(t)$ and the realized survivor index cash flow with $S(t, x)$ where "t" means the initially determined future time of payment exchange. In the current example both legs can be hedged but in a different way. An alternative hedge of the fixed leg can be achieved through the usage of zero-coupon fixed-income bonds. To hedge the floating leg a more complex method should be used as described below. To achieve this, the survivor index is approximated by expanding its cash flow. In particular, regarding the fixed legs of a series of q-forwards and their resulting net payoffs, the following approximation is made for the survivor index, presented in **Equation *1***, where:

- $\Delta(\mathrm{i}, \mathrm{x}+\mathrm{i}) = \mathrm{q}(\mathrm{i}, \mathrm{x}+\mathrm{i}) - \mathrm{q}_{\mathrm{F}}(0, \mathrm{i}, \mathrm{x}+\mathrm{i})$

- $\Delta(\mathrm{i}, \mathrm{x}+\mathrm{i})$ is the net payoff on the q-forward per unit at time $\mathrm{i}+1$
- $\mathrm{q_F}(0, \mathrm{i}, \mathrm{x}+\mathrm{i})$ = q-forward mortality rate at time „i" for age group „x+i", fixed at time 0

Equation 1, Approximation of a survivor index via a series of q-forwards - 1.

$$\mathrm{S}(\mathrm{t}, \mathrm{x}) = \big(1-\mathrm{q}(0, \mathrm{x})\big) \times \big(1-\mathrm{q}(1, \mathrm{x})\big) \times \ldots \times \big(1-\mathrm{q}(\mathrm{t}-1, \mathrm{x}+\mathrm{t}-1)\big) =$$

$$= \prod_{\mathrm{i}=0}^{\mathrm{t}-1}\big(1-\mathrm{q_F}(0, \mathrm{i}, \mathrm{x}+\mathrm{i})-\Delta(\mathrm{i}, \mathrm{x}+\mathrm{i})\big) \approx$$

$$\approx \prod_{\mathrm{i}=0}^{\mathrm{t}-1}\big(1-\mathrm{q_F}(0, \mathrm{i}, \mathrm{x}+\mathrm{i})\big) - \sum_{\mathrm{i}=0}^{\mathrm{t}-1} \Delta(\mathrm{i}, \mathrm{x}+\mathrm{i}) \prod_{\mathrm{i}=0, \mathrm{j} \neq \mathrm{i}}^{\mathrm{t}-1}(1-\mathrm{q_F}(0, \mathrm{j}, \mathrm{x}+\mathrm{j}))$$

Source: Carins et al. (2008, p. 108-109)

Continuing **Equation 1**, the floating leg $S(t,x)$ can be produced by holding the portfolio presented in **Equation 2**, where "r" means a constant interest rate.

Equation 2, Approximation of a survivor index via a series of q-forwards – 2.

- $-\frac{1}{(1+r)^{(t-1)}} \prod_{j=0, j \neq 1}^{t-1}\big(1-q_F(0, j, x+j)\big)$ units of the 1-year q-forward
- $-\frac{1}{(1+r)^{(t-2)}} \prod_{j=0, j \neq 1}^{t-1}\big(1-q_F(0, j, x+j)\big)$ units of the 2-year q-forward
- …
- $-\prod_{j=0, j \neq 1}^{t-1}\big(1-q_F(0, j, x+j)\big)$ units of the t-year q-forward

Source: Carins et al. (2008, p. 109)

During the calculation of the presented hedge quantities, it was implicitly assumed that, for example, the payment of the 1-year q-forward at time 1 is reinvested or „rolled up" until time t at the risk-free rate of interest. Therefore, all the payoffs are multiplied by an appropriate discount factor in order to be able to calculate the present values. For instance, the discount factor element for the 1-year q-forward is the $\frac{1}{(1+r)^{(t-1)}}$ (Carins et al., 2008).

Due to the stochastic environment, the determination of the quanto derivative is also a relevant step in **Equation 2**. Based on the definition of Cairns et al. (2008), quanto derivative is a financial instrument that delivers a certain number, "N", of a specified asset. The value of N is determined by a reference index that is distinct from the asset being delivered. Considering

Equation 1 and **Equation 2**, “N” equals $-\Delta(\mathrm{i}, \mathrm{x}+\mathrm{i}) \prod_{\mathrm{i}=0, \mathrm{j} \neq \mathrm{i}}^{\mathrm{t}-1}(1-\mathrm{q}_{\mathrm{F}}(0, \mathrm{j}, \mathrm{x}+\mathrm{j}))$, and N units of fixed-interest zero-coupon bonds are delivered at time $i+1$, maturing at time „t” with a price of $P(i+1, t)$ per unit at time $i+1$ (Carins et al., 2008).

Further elaborating on the recent example, it is essential to recognize the potential consequences of the lack of complete, real-world market regarding q-forward contracts. Because of the still developing nature of this segment of the capital market, hedge alternatives should be constructed from a narrower range of possible q-forward contracts. Such limitations may affect the range of available reference ages and maturities (e.g. maximum 20 years). Even though the presented example utilizes the assumption of a full market, it can still be used as a proper benchmark (Blake et al., 2019).

After outlining the fundamental characteristics of q-forwards, the introduction is concluded with a more readily understandable illustration, provided by Coughlan et al. (2007, p.2). Assume a 10-year q-forward contract with a reference population aligned to 65-year-old males in England & Wales. The participants of the contract are a pension plan (hedger) and a hedge provider. From the perspective of payments, the hedge provider pays a fixed rate which is proportional to a fixed mortality rate of 1.20%. On the other hand, the return payment from the pension plan is determined based on the value of the LifeMetrics Index for the specific subpopulation of males in England & Wales. The contract’s term sheet is presented in **Table 3**. Additionally, due to the existing 10 months delay in the availability of LifeMetrics data, the reference year used at the maturity of the contract is the index value in 2015.

Table 3, An illustrative term sheet for a single q-forward

Notional Amount	GBP 50,000,000
Trade Date	31 Dec 2006
Effective Date	31 Dec 2006
Maturity Date	31 Dec 2016
Reference year	2015
Fixed Rate	1.20%
Fixed Amount Payer	JPMorgan
Fixed Amount	Notional Amount x Fixed Rate x 100
Reference Rate	LifeMetrics graduated initial mortality rate for 65- year-old males in the reference year for England & Wales national population
Floating Amount Payer	XYZ Pension
Floating Amount	Notional Amount x Reference Rate x 100
Settlement	Net settlement = Fixed amount - Floating amount

Source: Coughlan et al. (2007, p. 3)

Considering the settlement calculation for the maturity of the contract, potential scenarios are presented in **Table 4**. If the reference rate in the reference year is lower than the fixed rate, then the net payment receiver is the pension plan, which is therefore able to compensate for its additional obligations emerging from the lower-than-anticipated mortality rates. On the contrary, if the reference rate in the reference year is higher than the fixed rate, then the pension plan is the one making payments, which it may cover from the decrease in the value of its liabilities. In **Table 4** positive settlement indicates that the pension plan pays, while negative settlement indicates that the pension plan receives payment (Coughlan et al., 2007).

Table 4, Possible settlement outcomes for the q-forward contract in Table 3

Reference Rate (Realized rate)	Fixed rate	Notional	Settlement
1.0000%	1.2000%	50,000,000	10,000,000
1.1000%	1.2000%	50,000,000	5,000,000
1.2000%	1.2000%	50,000,000	0
1.3000%	1.2000%	50,000,000	-5,000,000

Source: Coughlan et al. (2007, p. 3)

2.2.2.5 Longevity options

In the previous chapter, the essence of forward contracts was briefly presented: during a forward contract, two parties agree on a transaction where both the delivery date and price are initially fixed. The asset being delivered is called the underlying asset, which can vary from commodities to shares. In the case of forward contracts, there are two possible transaction positions: one party is selling (**long**), and the other party is buying (**short**). Both directions are binding once the contract is entered; the seller must sell even if it's not favourable on the delivery date, and the same applies to the buyer. Compared to forward contracts, options include the possibility of withdrawing, but only for one of the contract participants. Therefore, in case of options there are four possible positions for traders to take: right to purchase (**Long Call**), obligation to sell (**Short Call**), right to sell (**Long Put**) and obligation to purchase (**Short Put**). In both long call and long put options, the trader holds the right, but not the obligation, to purchase or sell the underlying asset if market conditions are favourable. However, the counterparty (short positions) in option contracts is obligated to fulfil its obligations regardless of its preferences. As a result, the obligated party is compensated with an up-front payment called premium, which can be regarded from the perspective of the long position holder as the price of the right to choose between buying or not buying (long call) and selling or not selling (long put) on the delivery date of the option contract. The delivery date of the option contract is called expiry date, which is an initially defined future date. In addition to expiry date, the specified price at which the predetermined amount of the financial asset can be purchased is also determined on the initiation of the contract. In formal terms, the predetermined amount called underlying, and the specified price is called exercise or strike price. European-style options can be exercised only on the date of expiry while American-style option can be exercised on the expiry date or even before it. Traders who seek flexible exercising without bearing the full cost of an American option have an alternative solution. Bermudan options allow for exercise on predetermined dates until the option's expiration, typically every once in a month (Chisholm, 2010).

To promote better understanding, consider the examples provided by Chisholm (2010, p.84 and 88). A call option contract is presented in **Table 5**, while a put option contract is presented in **Table 6**. Although the two tables are basically the same, they are presented separately to enhance clarity.

Table 5, Call option contract

Type of option:	American-style call
Underlying share:	XYZ
Number of shares:	100
Exercise price:	$100 per share
Expiry date:	One year from today
Current share price:	$100
Option premium:	$10 per share

Source: Chisholm (2010, p. 84)

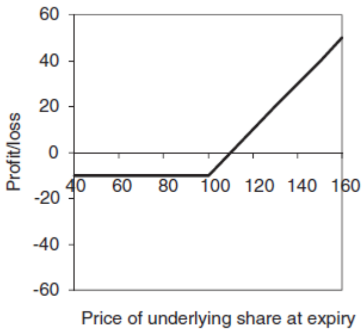
Table 6, Put option contract

Type of option:	American-style put
Underlying share:	XYZ
Number of shares:	100
Exercise price:	$100 per share
Expiry date:	One year from today
Current share price:	$100
Option premium:	$10 per share

Source: Chisholm (2010, p. 88)

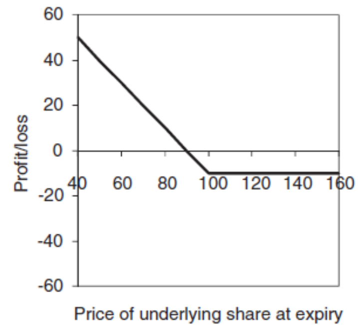
Furthermore, the expiry payoff profiles for all the four option positions are illustrated on **Figure 10**, **Figure 11**, **Figure 12** and **Figure 13** with additional explanations below the figures.

Figure 10, Expiry payoff for a long call



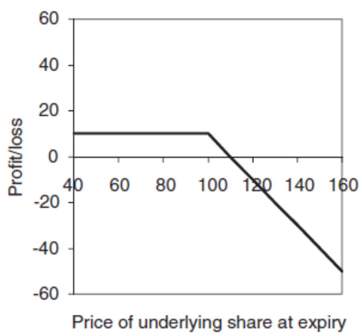
Source: Chisholm (2010, p. 86)

Figure 12, Expiry payoff for a long put



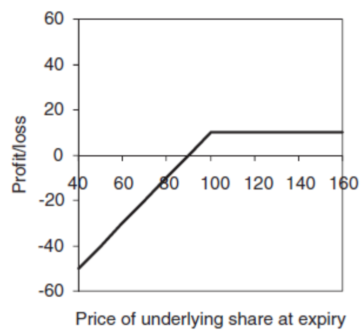
Source: Chisholm (2010, p. 89)

Figure 11, Expiry payoff for short call



Source: Chisholm (2010, p. 88)

Figure 13, Expiry payoff for a short put



Source: Chisholm (2010, p. 90)

In order to keep things simple and straight forward, the current explanation is ignoring transaction and funding costs. Similarly to the illustrations in **Figure 10**, **Figure 11**, **Figure 12** and **Figure 13**, the profit and loss profiles of the contracts are evaluated in terms of per-share basis on the expiry date. First consider the call contract. The holder of a long call option profits if the underlying share's value increases, while the seller of a short call option profits if the contract declared worthless and is not exercised. With respect to the profit and loss boundaries of the contract, the maximum loss of the long call option holder is $10 per share. This amount of loss is achieved if the underlying share's price is under the strike price and, therefore, the contract is not exercised. It basically means that the holder of the long call position only suffers the loss of the initial premium paid to the counterparty. On the other hand, if the underlying share's price increases, the potential profit of the trader in the long call position is technically infinite. Precisely, if the share's price is above the strike price at the date of expiry, then the option is exercised and the profit of the long call position holder is the share's actual price minus the strike price ($100) and the initial premium paid to the counterparty, which is $10 per share. It is important to mention that even if the contract is exercised it is not necessarily mean profit for the long call position holder. In the current example, the break-even point is at $110, which means that the trader only realizes profit if the share's value is above 110. If the share's value is between $100 and $110 then the initial loss of $10 per share is mitigated. For example, if the share is valued at $104 at the expiry date, then the contract is exercised, the shares are purchased at $100 (strike price) and then „immediately" sold for $104, seemingly realizing a profit of $4. However, due to the initial cost of the contract, a loss of $6 per share is realized ($4 - $10 = - $6). On the contrary, the maximum profit of the short call position holder is $10 if the contract expires worthless. Nevertheless, if the call option is exercised, the profit may turn negative, resulting in a loss for the short call position holder (Chisholm, 2010).

The calculation logic of a put options is similar to the call. Considering **Figure 12** and **Figure 13**, in light of **Table 6**, the trader in the long put position has the right to sell XYZ shares at the strike price of $100 each. Compared to the long call position, the long put position realizes profit if the share price remains below the strike price. Assume that the trader in the long put position does not own the shares but purchases them on the day of expiry when the option is exercised. In this case, if the price is $80 at the expiry date, the shares are purchased for $80 and then sold to the short put position holder at the strike price of $100. Therefore, the profit realized per share is calculated as follows: $100 (strike price received from long put position) - $80 (market purchase price) - $10 (initial premium) = $10. This also means that the profit is technically maximized for the long put position which is reached if the share's worth is $0. On the other hand, similarly to the long call position, the maximum achievable loss is $10 in form of the initial premium paid per share, if the option is not exercised. On the contrary, the maximum profit for the short put position is $10 if not exercised and -$100 (due to purchasing the stock at the strike price) + $10 (premium received) = -$90 per share if exercised at the share price of $0 (Chisholm ,2010).

Alternatively, traders may simultaneously enter into various call and put options to establish complex positions aimed at hedging specific risks (Chisholm ,2010). However, the introduction of these complex strategies is beyond the scope of the current thesis and therefore will not be explored in more detail.

Before changing the point of view from traditional options to longevity associated ones, it is vital to examine the motivation behind entering into option contracts. In contrast to contracts with linear payoff structures like forward derivatives, option contracts have non-linear payoffs, leading to significant differences. Generally, the non-linear payoff structure results from the asymmetric nature of options, where the potential gains or losses are not directly proportional to the underlying asset's price movement (Blake et al., 2006). Keeping in mind this feature of options, such contracts may prove useful for hedgers seeking to hedge against downside risk while leaving any upside potential at the same time. Besides, speculators may also take advantage of options if their intention is to trade volatility rather than levels of mortality rates (Carins et al., 2008). Broadly speaking, in finance history, the success of linear payoff derivatives has consistently encouraged development of option-based products as well (Dawson et al., 2009).

Following the description of Boyer and Stentoft (2013, p.38), the terminology of options designed to hedge longevity exposure can be characterized similarly to traditional options. In the simplest possible case, the strike price would be defined as the given price of a longevity risk exposure while other characteristics such as the date of the transaction (maturity date) would be defined identically to traditional options. Based on the authors' suggestion, the strike price can be defined as the expected survival rate (Boyer & Stentoft, 2013).

As options are complex derivatives with numerous potential applications, there are various approaches regarding their longevity alternatives as well. Longevity caps and longevity floors are both option-type longevity-linked derivatives which connect the buyer's payment to the survival rate of a reference portfolio. In case of caplets, the buyer receives the annual payment in the given year if the survival rate within a reference population cohort is more than the strike price established at the contract's inception. Conversely, the buyer receives payment from the floorlet if the reference population cohort is less than the strike price established at the contract's inception. Longevity caps can be decomposed into a series of sequentially maturing European style call options known as "caplets", which share a common underlying asset and a predetermined strike price. Similarly, longevity floors can be divided into a series of sequentially maturing European style put options known as 'floorlets,' which share an identical underlying asset and a prearranged strike price (Bravo & Nunes, 2021).

For a clearer point of view, the following example is examined regarding caplets and floorlets. In line with other longevity derivatives, the fundamental concept involves the usage of a survival index $S(t, x)$ as underlying asset. Denote the cap rate as $s_c(t)$ and the floor rate as $s_f(t)$ for exercise date "t". In this case the payment of the cap can be determined as $\max\{S(t,x) - s_c(t), 0\}$, while the floor's payment is based on $\max\{s_f(t) - S(t,x), 0\}$. By bundling together caplets and floorlets with corresponding properties (i.e., caplets with caplets and floorlets with floorlets), survivor caps and floors are created. Alternatively, survivor caps and floors are also often called longevity caps and floors (Blake et al., 2006, p. 182).

Another type of options like longevity risk management contracts are the mortality swaptions. Mortality swaptions are sophisticated contracts, involving a mortality swap as underlying "asset". This underlying mortality swap may vary by type and maturity, similarly to the swaption itself, which might be American, European or Bermudan in nature. Essentially, mortality swaptions provide the purchaser with the right to enter the swap from either position. More precisely, in case of a payer swaption, the holder has the right, but not the obligation, to enter as a fixed rate payer, while receiver swaption provide the right to enter as fixed-rate receiver. A payer swaption can be also regarded as a put position on survivor rates, as its value increases with a decline in survivor rates. From the perspective of a receiver swaption, its value increase depends on the rise of survivor rates, and therefore, it can be regarded as a call position on survivor rates. Although mortality swaptions are definitely useful from various risk management aspects, the ongoing development of a proper liquid market complicates the situation of such derivatives. Especially considering the valuation of these swaps on the exercise date with regards to the lack of benchmarks (Blake et al., 2006).

2.3 Mortality Models

Although the remarkable surge in human life expectancy experienced in the past decades enlarged the relevance of mortality models, models of such kind have central role for both institutional entities and private actors for a long time. While the primarily aim of insurers and reinsurers is to assess their capital requirements defined by economic or regulatory aspects, pension plans similarly apply mortality models essentially to evaluate uncertainty in funding levels. Additionally, subsequent to mortality rate improvements, stochastic mortality models have attained prominent position when comparing various solutions for managing longevity risk as well (Blake et al., 2006; Blake et al., 2019).

According to Blake et al. (2019), extrapolative or in other words time series mortality models can be classified as single-population or multi-population variants. Considering the first category, Lee and Carter (1992) laid the fundamentals of single population modelling with their groundbreaking methodology. As the present thesis extensively relies on the framework of the Lee-Carter model, a comprehensive explanation of the methodology is provided in the following sections. However, for the purpose of later comparisons it is important to emphasize that the traditional Lee-Carter model focuses merely on one factor to analyse the time series characteristics of longevity without making any assumptions regarding the degree of smoothness in mortality rates across adjacent ages or years (Blake et al., 2019; Boyer et al., 2014). As an extension of the original Lee-Carter framework, Cairns et al. (2006) introduced a more sophisticated approach by suggesting a second factor which impacts mortality dynamics to a greater extent at higher ages than in lower age groups. Taking into account that the original first factor reflects upon mortality-rate dynamics among all age groups identically, the second factor essentially helps differentiate among various age groups by utilizing an assumption of smoothness considering mortality rates across neighbouring ages within the same year (Blake et al., 2019; Boyer et al., 2014; Cairns et al., 2006). Besides the Lee-Carter and Cairns–Blake–Dowd (CBD) class models, the single-population category also consists of the P-splines model (Currie et al., 2004) and Age-Period-Cohort (APC) model (Osmond, 1985; Jacobsen et al., 2002) in accordance with the summarization provided by Blake et al. (2019). However, these models are not further elaborated upon in the present review.

In comparison with single population mortality models, multi population variants aim to enhance forecast quality by relying on an additional base population (Blake et al., 2019). As Blake et al., (2019) concludes, models of this sort are crucial for any entity which strives for proper longevity risk hedging via index-based instruments. The augmented common factor Lee Carter method was first proposed by Li and Lee (2005) and can be regarded as a significant milestone for multi population mortality models. The initial concept of the Li-Lee model seizes upon the inherent potential of similarities found in the historical mortality experience of population groups, while simultaneously recognizing their unique characteristics, including levels, age patterns, and trends (Li & Lee, 2005). Although the mortality rates in two populations may differ progressively, the principle of coherence embraces the simple fact that the ratio of mortality rates should not approach zero or infinity over time (Blake et al., 2019; Li & Lee, 2005). However, as both Villegas et al. (2017) and Enchev et al. (2017) pointed out, the Li-Lee model lacks stability regarding some actuarial applications, especially when stochastic evaluation of longevity risk management aspects come into account like basis risk measurement. For instance, Enchev et al. (2017) emphasizes that despite the sufficient fit of the Li-Lee model on different samples, problems arose with respect to the robustness and pace of convergence. On the other hand, Villegas et al. (2017) address issues regarding the length of historical data, specifically noting that in the absence of at least 10-12 years of reliable book data and without minimum annual exposure of 20,000–25,000 lives, the accuracy of two-population models becomes uncertain. Besides the Lee-Carter model, the Cairns–Blake–Dowd (CBD) model also got its multi-population extension by Li et al. (2015), within a decade after its initial introduction. Bearing in mind that Cairns et al. (2009) explored multiple potential adaptations of the original CBD model in the context of single-population, Li et al. (2015) proposed two-population variants for all these adaptations, facilitating a comprehensive comparison. Last but not least, multi-population variants were also developed for the other aforementioned single-population time series methods. While Cairns et al. (2011) and Dowd et al. (2011) provided the two-population version of the Age-Period-Cohort (APC) model, Biatat and Currie (2010) defined the two-population P-spline approach.

2.3.1 Lee-Carter model

In the present section, the previously briefly mentioned Lee-Carter (1992) method will be explored in more detail. As it was emphasized, Lee and Carter introduced a novel statistical procedure which revolutionised the way of modelling and forecasting mortality. The model focuses on the proper forecast of age-specific death rates by utilizing the following equation:

Equation 3, Lee-Carter model

$$ln\left({m^c}_{x,t}\right) = a_x + b_x \cdot k_t + \varepsilon_{x,t}$$

Source: Lee & Carter (1992, p.661)

where based on Lee and Miller's (2001) and Li and Hardy's (2011) explanation,

- $\boldsymbol{m^c}_{\boldsymbol{x,t}}$ represent the central rate of death at age „x” in year „t” for the modelled population
- $\boldsymbol{a_x}$ represents an age-specific parameter which explains the general age shape of the $m_{x,t}$ values, or in other words, reflecting the average mortality rate within the population at age „x”
- $\boldsymbol{b_x}$ represents another age-specific parameter which illustrates how the mortality tendency at age „x” changes in response to fluctuations in the general level of mortality (k_t), or in other words, it explains the sensitivity of $ln\left(m^c{}_{x,t}\right)$ to k_t
- $\boldsymbol{k_t}$ represents a time-varying index of the general level of mortality which reflects the overall pace of mortality improvements within the observed population
- $\boldsymbol{\varepsilon_{x,t}}$ represents the error term with 0 mean and $\sigma^2{}_t$ variance which expresses age-specific historical influences that are not accounted for by the model

Separately highlighting the b_x parameter, it provides essential information about the pace of decline of central death rates, particularly identifying those that decrease more rapidly and those that decline more modestly. In principle, b_x values may enter negative domains, suggesting that mortality tends to increase at particular ages, while decreasing at others. However, this phenomenon should diminish over the long run and b_x values should acquire identical signs for extended time intervals (Lee & Carter, 1992).

2.3.2 Forecasting with the Lee Carter model

Having introduced the model developed by Lee and Carter (1992), the next step is to further elaborate on its forecasting features. Taking advantage of the fact that the mortality index (k_t) values constitute a time-series dataset with one data point corresponding to each observed year, standard statistical approach can be employed to project this time-series (Lee & Miller, 2001). In fact, Lee and Carter (1992) came to the conclusion that the trajectory of the mortality indexes can be regarded as an Autoregressive Integrated Moving Average (ARIMA) process and was found to most closely resemble a Random Walk with Drift (RWD):

Equation 4, Lee – Carter model forecast, Random Walk with Drift

$$\boldsymbol{k_t = k_{t-1} + c + e_t \cdot \sigma \qquad e_t \sim N(0, 1)}$$

Source: Li et al. (2004, p. 22)

where based on the notation and description, provided by Li et al. (2004, p. 22),

- **c** represents the drift term, reflecting the linear trend in the evolution of k_t, which most of the times takes values from the negative domain
- $\boldsymbol{e_t \sigma}$ represents the deviation from the aforementioned linear change as random fluctuation which contributes to the generation of uncertainty during simulation processes

Bearing in mind that the presented forecast framework will serve as one of the cornerstones of the current thesis work, it is crucial to properly clarify the procedure and its associated characteristics. Hereinafter, this procedure will be briefly outlined from the perspective of Li et al. (2004, p.22). Taking into account that the differences of $k_t - k_{t-1}$ presumed to be independent and identically distributed variabless, the estimates of the mean (c) and standard deviation (σ) can be captured by the **Equation 5 and 6** (Li et al., 2004).

Equation 5 and Equation 6, Estimates of the mean (c) and standard deviation ($\boldsymbol{\sigma}$)

$$\hat{c} = \frac{1}{T}\sum_{t=1}^{T}(k_t - k_{t-1}) = \frac{k_T - k_0}{T}$$

$$\hat{\sigma} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(k_t - k_{t-1} - \hat{c})^2}$$

Source: Li et al. (2004, p. 22)

while the standard error of the estimated mean (c) can be defined as presented in **Equation 7**:

Equation 7, Standard error of the estimated mean (c)

$$\sqrt{\mathrm{var}(\hat{c})} = \sqrt{\frac{\sigma^2}{T}} \approx \frac{\hat{\sigma}}{\sqrt{T}}$$

Source: Li et al. (2004, p. 22)

The characterization of the standard error of the estimated mean (c) is necessary since $\hat{c}$ depicts a sample value which may vary across unique samples. In other words, various realizations of historical ${m^c}_{x,t}$ values result in different k_T samples, leading to varying estimates of mean ($\hat{c}$) values (Li et al., 2004). Considering that the e_t component in the deviation from the linear change conforms to Gaussian distribution, $\hat{c}$ can also be stated in the manner of **Equation 8**, where η represents a standard-normal random variable.

Equation 8 Alternative form of the estimates of the mean ($\hat{c}$)

$$\mathbf{\hat{c} = c + \sqrt{var(\hat{c})} \cdot \eta}$$

Source: Li et al. (2004, p. 22)

By utilizing the overview of forecasting characteristics so far, the path is clear to extrapolate the general levels of mortality change (k_t). For this to happen, it is required to select a narrow range of η based on the associated probability, along with a series of sample values of e_s that are independent of η for s = (T + 1) to „t" (Li et al., 2004).

Equation 9, Lee – Carter forecast, an alternate to Equation 4.

$$\mathbf{k_t = k_T + \left(\hat{c} - \sqrt{var(\hat{c})} \cdot \eta\right) \cdot (t - T) + \hat{\sigma} \cdot \sum_{s=T+1}^{t} e_s}$$

Source: Li et al. (2004, p. 23)

The general levels of mortality change (k_t) can be expressed even despite the lack of knowledge about the exact drift term (c), as the probability for c spans any interval defined by **Equation 8**, which crucial information is integrated into the outlined forecasting process (**Equation 9**) via the simulation of η (Li et al., 2004). Mentioning the dependence of the simulated future levels of mortality change (k_t) is also relevant. According to Li et al. (2004), the particular trajectory depends on three main aspects such as the estimated drift (ĉ), the randomly generated disparity between the true mean (c) and the estimate (ĉ) and last but not least on the random innovations.

2.3.3 Relationship between $m_{x,t}$ and $q_{x,t}$ values

As $m_{x,t}$ values constitute a vital function in the model, it is deemed necessary to present their way of calculation in more detail. Considering the explanations provided by Cairns et. al. (2009) and Kim and Choi (2011), the **Equation 10** illustrates the estimation process, where:

- $D_{x,t}$ represents the number of deaths measured at age „x" during year „t"
- $E_{x,t}$ represents the central exposure to risk which indicates the average population size for age group „x" during calendar year of „t"

Equation 10, Central rate of death estimation

$$\boldsymbol{m^c}_{\boldsymbol{x,t}} = \frac{\boldsymbol{D_{x,t}}}{\boldsymbol{E_{x,t}}}$$

Source: Cairns et. al. (2009, p.2)

Following the approach of Carins et al. (2009), an estimate of the original population is conventionally used to approximate the average population. The computation of the estimate is based on the composition of individuals aged „x" as of their last birthday, occurring at the midpoint of the calendar year. Iy this way, the Authors define the underlying death rate ($m^c{}_{x,t}$) which should be identical with the ratio of deaths and exposures (Carins et al., 2009).

Given that one of the most widely considered measure of mortality is the mortality rate ($q_{x,t}$), it is essential to demonstrate its connection with the central rate of death values. The mortality rate fundamentally captures the probability that an individual who has attained the age „x" at time „t" will not survive to celebrate their subsequent birthday in „t+1" (Carins et al., 2009). As Vékás (2016) illustrates, mortality rates can be expressed via the utilization of **Equation 11**:

Equation 11, Definition of mortality rates ($\boldsymbol{q_x}$)

$$\boldsymbol{q_x} = \mathbf{P}(\boldsymbol{L} < \boldsymbol{x} + \mathbf{1} \mid \boldsymbol{L} \geq \boldsymbol{x}) \quad (\boldsymbol{x} \in \boldsymbol{N})$$

Source: Vékás (2016, p.55)

where „L" represents a non-negative random variable measured in years, called "lifetime". The relationship between $m^c{}_{x,t}$ and $q_{x,t}$ values is described by **Equation 12** (Carins et al., 2009).

Equation 12, Relation between mortality rates and central death rates

$$\boldsymbol{q_{x,t}} = \mathbf{1} - \boldsymbol{e}^{-\boldsymbol{m^c}_{\boldsymbol{x,t}}}$$

Source: Cairns et. al. (2009, p.3)

3 Research question and hypotheses

After the overview of the theoretical background of longevity risk management, the aim of the current section is to present the research question and related hypotheses.

The present thesis has several fairly related goals to explore. Its fundamental aim is to overview the longevity phenomenon, with a strong emphasis on exploring insurance-based and capital market-based risk management solutions. Beyond this goal, its research objective is to gain a better understanding of the net return on Longevity Reinsurance transactions (chapter 2.2.1.3) from the perspective of the reinsurer. Especially, considering the distribution of the reinsurance contract's return when comparing populations of different age groups via simulation. Therefore, the main research question of the current thesis can be defined as follows:

What is the impact of demographic factors, such as age groups, on the return of Longevity Reinsurance contracts from the perspective of reinsurers?

The hypotheses formulated to aid in addressing the research question:

H. Main: The distribution of the return of Longevity Reinsurance contract is significantly dependent on the age composition of the reinsured portfolio**.**

- **H1:** The distribution of the return significantly differs depending on whether the underlying portfolio consists of individuals aged **60-70**, or individuals aged **70-80**.
- **H2:** The distribution of the return significantly differs depending on whether the underlying portfolio consists of individuals aged **60-70**, or individuals aged **80-90**.
- **H3:** The distribution of the return significantly differs depending on whether the underlying portfolio consists of individuals aged **70-80**, or individuals aged **80-90**.

4 Methodology

To address both the research question and hypotheses, several preparatory steps had to be made. It is important to emphasize that all the preparatory and modelling tasks were executed using R programming language (version 4.3.3).

As an initial step, a comprehensive data collection took place focusing on parameters required to estimate central rate of death ($\boldsymbol{m^c}_{\boldsymbol{x,t}}$) for male, female, and unisex cases, as **Equation 10** presents. This step was necessary because the built-in Lee-Carter modelling function (from the demography package, lca function) required this parameter, in addition to central exposures to risk ($\boldsymbol{E_{x,t}}$) values. While the raw data collected from Human Mortality Database (mortality.org, 2023) consisted of a wider age range and years, the age group under observation was limited to 0-100 years, and the year span was set as 1966-2020. Therefore, 55 years served as the basis for the latter mortality rate forecasts which is in line with the original Lee-Carter (1992) article where the authors used similar year span (1933-1987). Additionally, this approach also excluded the temporary mortality rise due to the recent COVID-19 pandemic which otherwise could potentially bias the long-term mortality forecasts. The data collection was carried out for 17 countries but for reasons such as completely missing years and data within the basis years span (1966-2020), 6 countries were excluded. The countries appropriate for the pre-set year span condition were Belgium, Canada, Czech Republic, France, Hungary, Japan, Norway, Sweden, Switzerland, UK and USA. Although these countries were all suitable for the current research, only Hungary was considered in the final setup because of capacity limitations. However, this leaves room for further research.

After the determination of central rate of death values ($\boldsymbol{m^c}_{\boldsymbol{x,t}}$) with the „x" index ranging from 0 to 100 ages and the „t" index ranging from 1966 to 2020 years, Lee-Carter modelling was applied. By utilizing the results, the forecasting of the time-varying index of mortality level ($\boldsymbol{k_t}$) took place. As presented by the Lee-Carter model in **Equation 3**, this is the only time dependent parameter which reflects the overall pace of mortality improvements within the observed population. With the help of the forecasted $\mathbf{k_t}$ parameter values, future mortality rates ($\mathbf{q_{x,t}}$) could be determined. This can be achieved by first calculating central rate of death ($\mathbf{m^c}_{\mathbf{x,t}}$) for future years based on **Equation 3**, then by utilizing **Equation 12** with the calculated $\boldsymbol{m^c}_{\boldsymbol{x,t}}$

values across the observed age span in the future years. The forecast took place for 3 + 40 years, where the first 3 years was necessary to reach the hypothetical start year (current year – 2023) of the Longevity Reinsurance contract, and 40 years to reach the predetermined boundary of the observation period. Although a longer observation period might be feasible, the associated increase in forecast uncertainty was deemed undesirable. Furthermore, the forecast was implemented in two streams. Firstly, the best estimates for each gender case were determined by utilizing the built-in "forecast" function from the demography package. Then an additional 10,000 trajectories were generated via a manually built function following the approach presented in chapter 2.3.2.

The R script used during the simulation of trajectories is presented in **Appendix 1** and **Appendix 2**. The main input objectives were two lists, containing the estimated central death rates and exposure rates in data frame structure per country.

Both the best estimate and additional trajectories were utilized during the calculation of the Longevity Reinsurance contract. After the generation of a reference populations, consisting of 1.000 people within the age spans predetermined during the hypotheses setting (e.g., 60-70 years old), immediate life-time annuities were calculated based on one-time premiums. Then the best estimate trajectories were used to determine the fixed leg of the reinsurance transaction considering the 40-year observation period. Similarly, the 10.000 additional trajectories were used to calculate 10.000 floating legs, thereby forming a payout distribution based on various mortality scenarios. The relationship between the fixed leg and the floating leg distributions proved to be a proper foundation, offering sufficient answers for both the research question and hypotheses. The relations were also perceived in cases where the reinsurer applied 1% or 2% margin on the fixed leg, to ensure the profitability of the transaction.

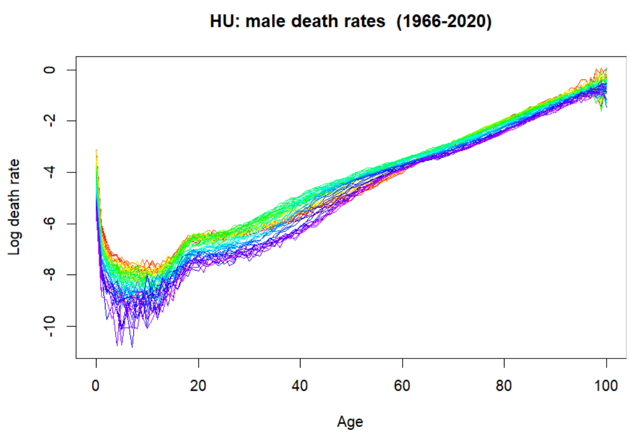
In the following subsections the forecasting and modelling steps will be presented in greater detail.

4.1 Lee-Carter modelling in practice

The Lee-Carter modelling calculations were performed using the built-in function "lca" from the "demography" package. However, before its application, support objects needed to be created based on the central rate of death $(\boldsymbol{m^c}_{\boldsymbol{x,t}})$ and the central exposures to risk $(\boldsymbol{E_{x,t}})$ values.

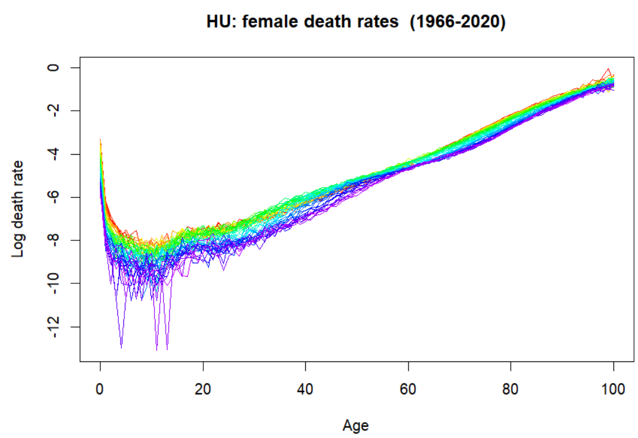
This goal was accomplished using the "demogdata" function from the same "demography" package. This intermediate step also left room for the visualization of the development of the central rate of death values, as illustrated in **Figure 14** (Male), **Figure 15** (Unisex) and **Figure 16** (Female).

Figure 14, Log central rate of death development – Hungary (Male)



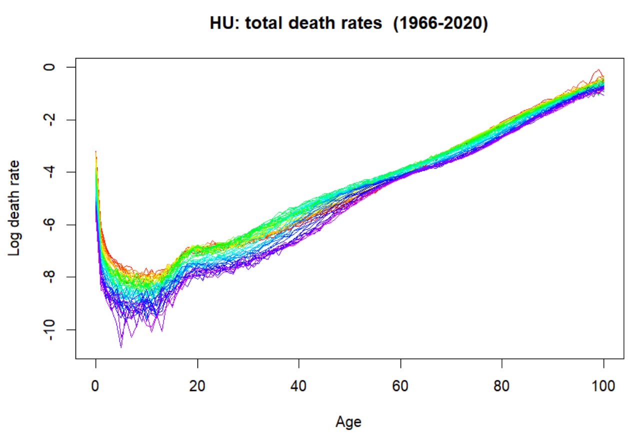
Source: Own calculations

Figure 16, Log central rate of death development – Hungary (Female)



Source: Own calculations

Figure 15, Log central rate of death development – Hungary (Unisex)



Source: Own calculations

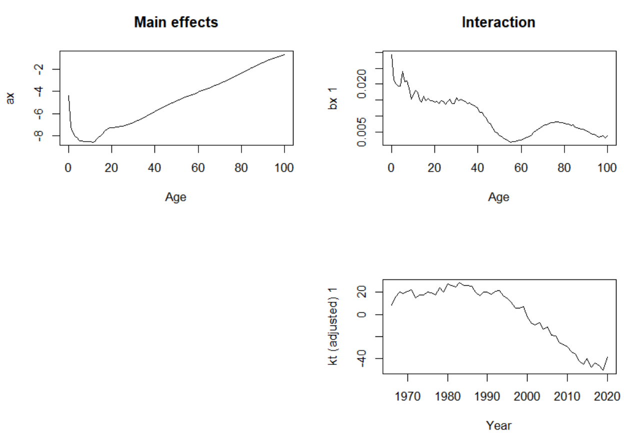
In **Figure 14-16,** each lines illustrate a given year, between 1966-2020. While the red and yellow lines indicate closer years to the present, blue and purple lines indicate years in the 60's 70's. Therefore, the mortality improvements described in chapter 2.1. can be clearly observed.

By utilizing the support objects, the Lee Carter modelling could be accomplished for the countries under observation. The evolution of the Lee-Carter parameters across ages ($\boldsymbol{a_x}$ and $\boldsymbol{b_x}$) and years ($\boldsymbol{k_t}$) is illustrated on **Figure 17** (Male), **Figure 18** (Unisex) **Figure 19** (Female). The interpretation of each parameter can be found at the description of **Equation 3**.

Figure 17, Evolution of Lee-Carter model parameters – Hungary (Male)

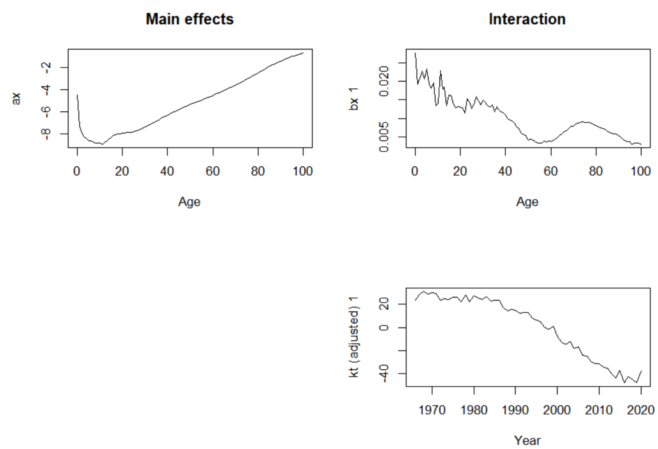
Source: Own calculations

Figure 18, Evolution of Lee-Carter model parameters – Hungary (Unisex)



Source: Own calculations

Figure 19, Evolution of Lee-Carter model parameters – Hungary (Female)

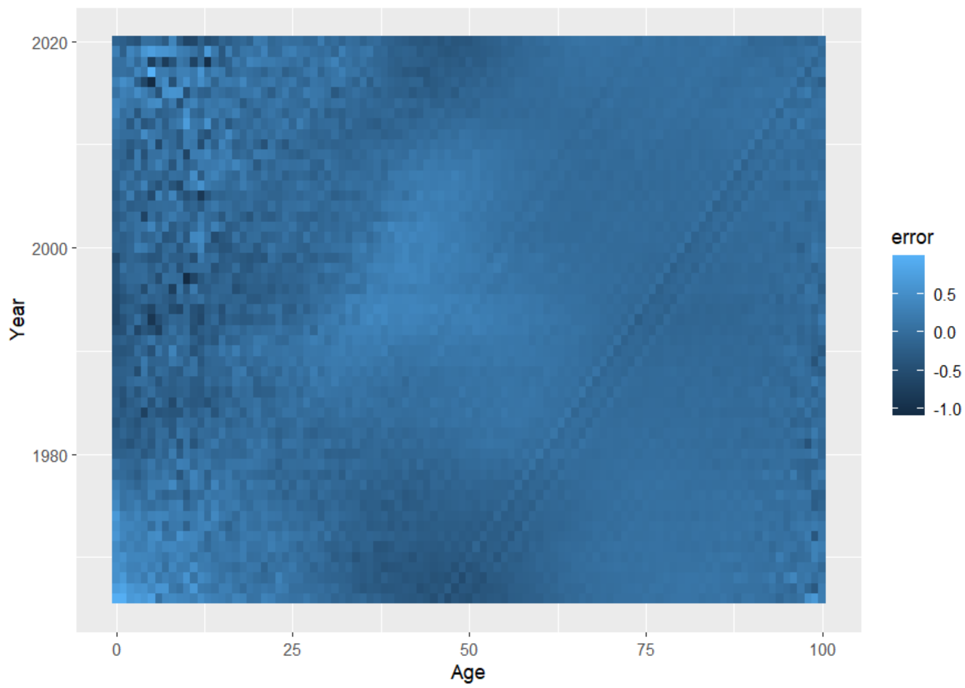


Source: Own calculations

Despite the similarity in parameter evolution for the different gender cases, differences can be observed in nearly every aspect. These discrepancies might seem minor, but they have significant impact on the development of mortality rates, which effect was clearly observable during the forecasting stage of the current research.

Besides the development of the central death rates and the evolution of Lee-Carter parameters, the error of the model can also be presented, as illustrated in **Figure 20**. In **Figure 20**, only the unisex case is showcased, but the same interpretation could apply to males and females as well.

Figure 20, Lee-Carter model errors across age groups and years – Hungary (Unisex)



Source: Own calculations

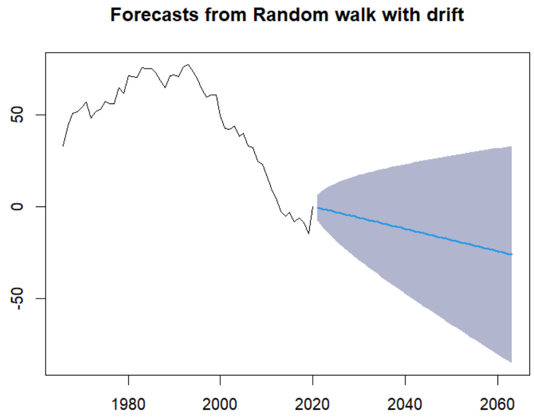
In case the error is positive, then the model underestimates mortality while overestimates it if the error is negative. It is important to interpret the meaning of the emerging diagonals which always represent a cohort of people. The significantly different mortality development observed in these cohorts compared to others may be attributed to outstanding events, such as World War I. Researchers found that children who lived through this great tragedy had weaker immune systems due to, for instance, malnutrition. This resulted in higher mortality rates within the age cohort throughout the following decades (Allais et al., 2021; Elo & Preston, 1992).

4.2 Lee-Carter forecasting in practice

Forecasting was conducted based on the calculated Lee Carter models, as anticipated in the brief overview provided in chapter 4. The forecasting procedures aimed on the time-varying index of mortality level ($\boldsymbol{k_t}$) parameter from **Equation 3** which is the only time dependent parameter, reflecting the overall pace of mortality improvements within the observed population. The development of this parameter throughout the observed year span (1966-2020) for Hungary is illustrated in **Figure 17-19.** The forecast spanned 43 years, with 3 years to reach the hypothetical start year of the Longevity Reinsurance contract (current year – 2023) from 2020, followed by a future observation period of 40 years. The forecast span was determined to be neither too small, which could limit accuracy, nor too large, which could increase forecast uncertainty significantly.

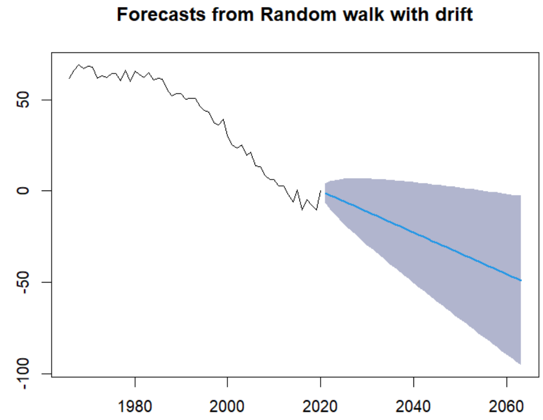
As a first step the Best Estimate trajectories were calculated for each gender cases by utilizing the built-in „forecast" function from the demography package. While other forecasting approaches could also prove to be efficient, the built-in „forecast" function uses Random Walk with Drift as proposed by Lee and Carter (1992), presented in **Equation 4**. The Best Estimate trajectories were intended to represent the most likely outcome of $\boldsymbol{k_t}$ parameter value development over the forecasted 43 years. As an illustration, **Figure 21** present the male, **Figure 22** present the female and **Figure 23** presents the unisex Best Estimate $\boldsymbol{k_t}$ trajectories with 95% confidence intervals. It is relevant to mention that the used „forecast" function organizes the historical $\boldsymbol{k_t}$ parameter values in such way that the last observed parameter ($\boldsymbol{k_T}$) is set to 0. This is achieved by subtracting the $\boldsymbol{k_T}$ value from historical $\boldsymbol{k_t}$ values. Although this adjustment does not affect the forecast itself, it is the cause behind the slight visual differences between **Figure 17-19** and **Figure 21-23.**

Figure 21, Best Estimate k_t forecast (Male)



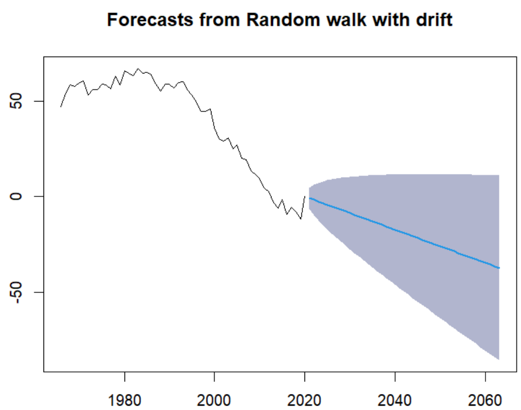
Source: Own calculations

Figure 23 Best Estimate k_t forecast (Female)



Source: Own calculations

Figure 22 Best Estimate k_t forecast (Unisex)



Source: Own calculations

After determining the Best Estimate Trajectories, the next step before proceeding with the direct Longevity Reinsurance preparations was to calculate additional trajectories. While the Best Estimates were created for the fixed leg calculation, the purpose of the additional trajectories was to serve as the basis for the floating leg calculations. To ensure greater accuracy, 10,000 trajectories were generated for each gender case using a manually built function following the approach presented in chapter 2.3.2. The results of the different parameter estimations required for the forecasts are summarized in **Table 7** by gender.

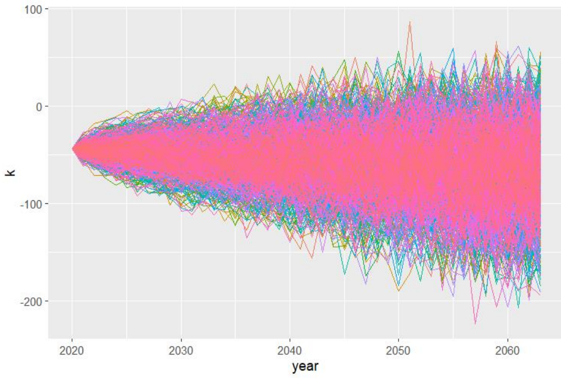
Table 7, Lee Carter forecast – manually built function parameter estimations (Hungary)

Name	Notation	Equation number	Value (Male)	Value (Female)	Value (Unisex)
Drift mean estimate	$\hat{c}$	Equation 5	-0.5976	-1.1189	-0.8507
Estimate of the standard deviation	$\hat{\sigma}$	Equation $5$6	5.1488	4.0391	4.2080
Standard error the drift mean estimate	$\sqrt{\mathbf{var}(\hat{\mathbf{c}})}$	Equation 7	0.6942	0.5446	0.5674

Source: Own calculations

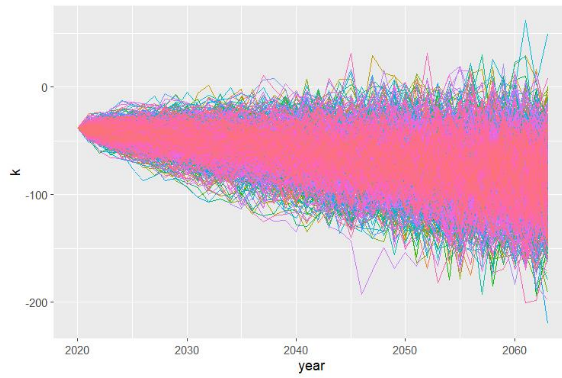
As **Table 7** presents, the estimates of the linear trend in the evolution of $\boldsymbol{k_t}$ values are negative in all gender cases. In the unisex case for instance, this means that the general level of mortality decreases by an average of -0.8507 per year. This is also in line with chapter 2.1, where the increasing life expectancy was elaborated upon from multiple aspects. While the rate of decline was the highest among females (-1.1189), the estimate's standard error stood out in case of males (0.6942). Additionally, the standard deviation from the linear change in the general level of mortality development was estimated to be the highest among males as well (5.1488). Based on the parameter estimations, the extrapolation of the general levels of mortality change (k_t) could be carried out via **Equation 9**. As illustration of the extrapolation, **Figure 24** presents 1000 out of the 10.000 additional male trajectories, while **Figure 25** presents the development of 1000 female trajectories over the forecasted 40 years. **Figure 24-25** are clearly consistent with the trends presented in **Table 7**, particularly reflecting the greater decrease in the general level of mortality among females. Moreover, the larger standard deviation is also evident in case of males.

Figure 24, Additional male trajectories – Hungary (Manually built function)



Source: Own calculations

Figure 25, Additional female trajectories – Hungary (Manually built function)



Source: Own calculation

At this point, having both the Best Estimate trajectories and additional trajectories of the time-varying index of the general level of mortality ($\boldsymbol{k_t}$), it was possible to proceed with the future mortality rate ($\boldsymbol{q_{x,t}}$) calculations. By utilizing the forecasted $\boldsymbol{k_t}$ values, the central rate of death ($\boldsymbol{m^c{}_{x,t}}$) values were calculated for the future years based on **Equation 3**. Following this, the application of **Equation 12** ensured that the future mortality rates ($\boldsymbol{q_{x,t}}$) were available for the Longevity Reinsurance leg calculations. However, prior to the subsequent steps, a brief check took place. To confirm that the mortality rates sourced from the Best Estimate trajectory are indeed the best estimates, the mortality rates sourced from the additional 10.000 simulations were compared to them. In fact, if the best estimate trajectory is correct, then its values should differ minimally from the averages of the additional trajectories. To inspect this connection, the age group 65 and the future year of 2063 were chosen. The statistical measures of the distribution formed from the values of the 10.000 simulations regarding age group 65 in year 2063 are presented in **Table 8**. Considering **Table 8**, the relationship between the Best Estimate mortality rates and the simulated mortality rates appears to be correct. The mean of the 10.000 trajectories can be regarded as particularly similar, with only minor differences. Furthermore, nearly half of the trajectories fall below and half fall above the Best Estimate trajectory, indicating that the Best Estimate is indeed a proper estimate.

Table 8, Comparison of Best Estimate and simulated mortality rates $(q_{x,t})$

Hungary, Age group 65, year 2063

	Unisex	Male	Female
Best Estimate mortality rate value (BE)	0.0181352	0.0286392	0.0101693
Mean mortality rate of trajectories (10.000)	0.0183698	0.028900	0.0104012
Maximum mortality rate of trajectories (10.000)	0.0307240	0.0489891	0.0218144
Minimum mortality rate of trajectories (10.000)	0.0103939	0.0162228	0.0048109
Number of higher than BE trajectories	5033	5050	5050
Number of lower than BE trajectories	4967	4950	4950

Source: Own calculations

4.3 Calculation of reference population parameters

Before delving into the specific Longevity Reinsurance calculations by utilizing both the Best Estimate trajectories and additional trajectories, it was relevant to define a reference population. Similarly to the consideration of the number of additionally simulated trajectories, a reference population with 1000 individuals was defined for each relevant age categories. In the case of the present research, it meant one population consisting of individuals with ages ranging from 60 to 70, one population with age range of 70 to 80 and a last population with age range of 80-90. In this section, only the population with the age range of 60-70 is presented in more detail. Nevertheless, the additional two populations followed the same characteristics and are compared in chapter 5.

During the reference population simulations three main aspects were determined for each individual of the population: age, gender, and the one-time premium paid. Considering the age and gender parameters, these were simulated based on uniform distribution. Therefore, the number of simulated males was 489 and the number of simulated females was 511. The distribution of the age groups is presented in **Appendix 3**. Besides age and gender, the one-time premium paid was necessary to define the immediate life-time annuities „purchased" by the population members. These life-time annuities play an essential role during the Longevity Reinsurance modelling because these represent the reinsured obligations in 40 years' time. More precisely, the valuation of the transaction legs is determined by calculating the sum product of each population member's annual life annuity payout and their survival rates in the observed years, which are then discounted to present value. If the survival rates are based on the Best Estimate, then the fixed leg is calculated, while each additionally simulated trajectory results a floating leg. This calculation is presented in more detail in chapter 4.4.

Following the approach of three analysed articles, it is observed that the income distribution of the majority of populations typically closely resembles an exponential pattern especially for low- and middle-income classes (Drăgulescu & Yakovenko, 2001; Bogdan et al., 2017; Tao et. al., 2019). Therefore, exponential distribution ($\boldsymbol{\lambda \sim Exp}$) was used during the calculation of one-time premiums paid. In the case of the exponential distribution, there is a strict connection between its parameter and the distribution's first moment, or in other words, its expected value. This connection is presented in **Equation 13**.

Equation 13 Mean of exponential distribution

$$\boldsymbol{E(X) = \frac{1}{\lambda}}$$

Source: Michaletzky (2016, p. 155)

To calculate the $\boldsymbol{\lambda}$ parameter for Hungary, the median of USA retirement saving account in 2022 ($87.000) was used as a basis (Federal Reserves, 2023). Additionally, direct proportionality was used to calculate the median of Hungarian retirement savings accounts in 2022 based on the average annual wages (OECD, 2023). The brief calculation is presented in **Appendix 4**. Considering a necessary lower boundary for the purchase of immediate life-time annuities, a minimum one-time premium of $10,000 was established. This amount is equivalent to approximately 3,661,700 Ft (MNB, 2024), which was found to be sufficient amount for a modest investment to supplement pension payments.

Drawing from the one-time premiums paid by each member of the reference population, the immediate life-time annuities could be calculated. For this objective, actuarial estimation concepts were utilized. However, prior to the actual calculations, it is essential establish some actuarial foundations. Actuarial pricing calculations are derived from life tables consisting of several parameters. The main parameters are the mortality rates ($\boldsymbol{q_{x,t}}$) for each age group from 0 to 100. Mortality rates fundamentally captures the probability that an individual who has attained age „x” at time „t” will not survive to celebrate their subsequent birthday in „t+1” (Carins et al., 2009; Vékás, 2016). As a complement to mortality rates, survival rates can be easily estimated using **Equation 14**, representing the probability of an individual surviving to age „x+1”, having survived to age „x” (Banyár, 2021, p. 21).

Equation 14, Calculation of survival rates ($\boldsymbol{p_{x,t}}$)

$$\boldsymbol{p_{x,t} = 1 - q_{x,t}}$$

Source: Banyár (2021, p. 21)

Based on the mortality and survival rates, the number of survivors ($\boldsymbol{l_{x,t}}$) in each group can be calculated. Starting as 100,000 people ($\boldsymbol{l_{0,t} = 100.000}$), the number of survivors is calculated using **Equation 15**. Similarly, **Equation 16** sheds light on the calculation of the number of deaths ($\boldsymbol{d_{x,t}}$) at given ages.

Equation 15, Calculation of the number of survivors ($\boldsymbol{l_{x,t}}$)

$$\boldsymbol{l_{x+1,t} = p_{x,t} \cdot l_{x,t}}$$

Source: Banyár (2021, p. 29)

Equation 16, Calculation of the number of deaths ($\boldsymbol{d_{x,t}}$)

$$\boldsymbol{d_{x,t} = l_{x,t} - l_{x+1,t}}$$

Source: Banyár (2021, p. 170)

By considering both the number of survivors and the number of deaths, commutation numbers can be obtained. Commutation numbers are standard functions obtained from life tables, commonly used in actuarial science. **Equation 17** and **Equation 18** showcases the calculation of the „discounted value” of living **($D_{x,t}$)** and death **($C_{x,t}$)**.

Equation 17, Calculation if the discounted value of living

$$D_{x,t} = l_{x,t} \cdot v^{x}$$

Source: Banyár (2021, p. 171)

Equation 18, Calculation if the discounted value of death

$$C_{x,t} = d_{x,t} \cdot v^{x+1}$$

Source: Banyár (2021, p. 171)

where:

- „v**”** represents the discount factor, calculated as $v = \frac{1}{1+i}$,
- „**i**” represents the technical interest rate

The technical interest („i”) rate during the Hungarian pricing process was determined based on the December 2023 base rate of the Hungarian National Bank which was 10.75% (Statista, 2023). Utilizing the commutation numbers, the calculation of immediate life-time annuities with 1 Ft. annual annuity payment could be conducted, as demonstrated by **Equation 19**.

Equation 19, Calculation of immediate life-time annuities (1)

$$\ddot{a}_x = \frac{N_{x,t}}{D_{x,t}}$$

Source: Banyár (2021, p. 181)

where:

- $N_{x,t}$ represents the sum of the discounted value of living, calculated as $N_{x,t} = D_{x,t} + D_{x+1,t} + D_{x+2,t} + \cdots + D_{\omega,t}$
- ω represents the highest shown age level
- $\ddot{a}_x$ represents the one-time premium paid by the purchaser (single net premium)
- $\frac{N_{x,t}}{D_{x,t}}$ is typically referred to as "annuity factor"

Equation 19 can be rearranged to directly express the annual payouts, given that in our case, the one-time premiums are predetermined parameter rather than estimated ones. **Equation 20** presents the rearranged form of **Equation 19**.

Equation 20, Calculation of immediate life-time annuities (2)

$$\boldsymbol{SA} = \boldsymbol{\ddot{a}_x} : \frac{\boldsymbol{N_{x,t}}}{\boldsymbol{D_{x,t}}}$$

Source: Banyár (2021, p. 181)

where:

- SA represents the „sum assured”, which in the current case indicates the annual payout of the life-time annuity.

During the calculation of the annual payments of the immediate life-time annuities, the Unisex Best Estimate Trajectory was used in line with the Gender Directive, which forbids pricing discrimination based on gender (European Union, 2004). The pricing process involved 101 mortality rates ($\boldsymbol{q_{x,t}}$) ranging from age 0 to age 100. Although theoretically, the range could have been extended to age 110, it had no impact on the results since $\boldsymbol{D_{x,t}}$ depends solely on the annuity’s start age (entry age in our case) and $\boldsymbol{N_{x,t}}$ values are negligible for ages 100 to 110.

4.4 Longevity Reinsurance in practice

The completion of the reference population calculations meant that the last pieces of the Longevity Reinsurance preparatory steps have fallen into their places. Hence, the estimation of the fixed and floating legs could take place. As it was briefly overviewed in chapter 4, the mortality rates ($\boldsymbol{q_{x,t}}$) sourced from the Best Estimate trajectories served as the basis for the fixed leg calculations, while the additionally simulated 10.000 trajectories served as the basis for the floating leg calculations. For further details regarding the concept of Longevity Reinsurance alongside with the role of the transaction „legs”, please refer to chapters 2.1.1 and 2.2.1.3.

Although the used trajectories were different, the estimation steps were similar in the cases of both the fixed and floating legs. As it was briefly mentioned in chapter 4.3, the valuation of the transaction legs was determined by calculating the sum product of each population member's annual life annuity payout and their survival rates in the observed years, which are then discounted to present value. To be precise, **Equation 21** presents the calculation formula of floating leg „i”.

Equation 21, Calculation formula for Longevity Reinsurance transaction legs

$$\boldsymbol{V_i} = \sum_{\boldsymbol{j=1}}^{\boldsymbol{1000}} \sum_{\boldsymbol{t=2023}}^{\boldsymbol{2063}} \sum\nolimits_{\boldsymbol{x}} \boldsymbol{SA_j(x,t) \cdot p_j(x,t) \cdot v(t)}$$

Source: Own calculations

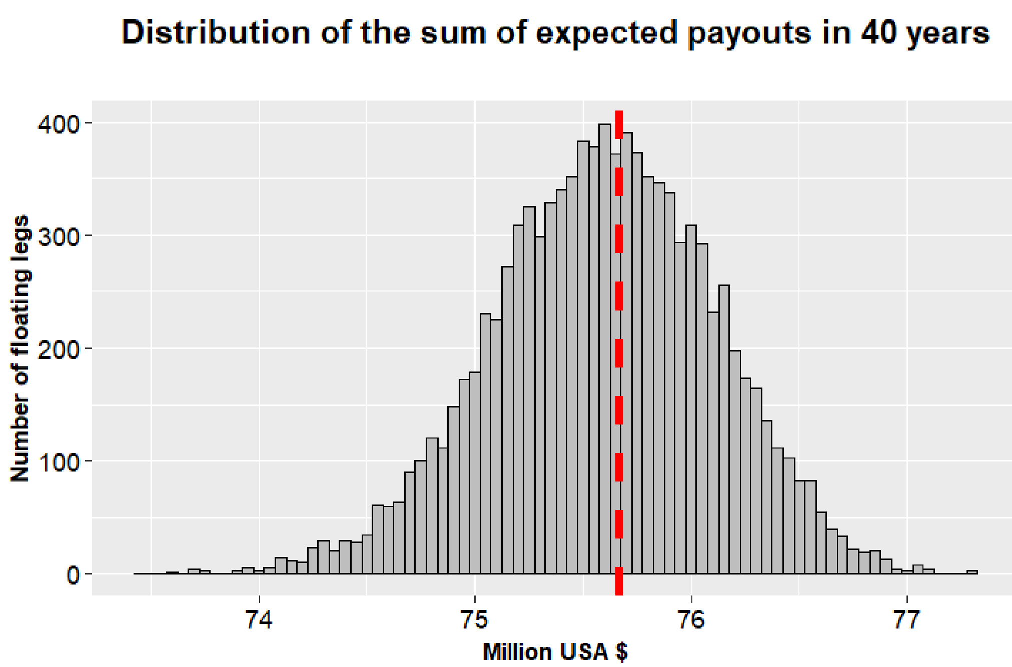
where:

- „V_i” represents the valuation of the i^{th} floating leg
- „j” iterates over in the reference population of 1.000 individuals
- „t” iterates over the forecasted years from 2023 to 2063
- „x” represents the age of the person „j” in year „t”
- $SA_j(x,t)$ represents the annual life annuity payout for person „j” aged „x” at time „t” (This amount remains constant for person „j” across all ages „x” and times „t”)
- $p_j(x,t)$ represents the survival probability of person „j” aged „x” at time „t”
- $v(t)$ represents the discount factor for the year „t”, used to discount future cash flows to their present value

Based on the simulated trajectories, index „i” runs from 1 to 10.000. As **Equation 21** illustrates, $\boldsymbol{p_j(x,t)}$ is the equation part, which is affected by the development of the mortality, or in other words, by the simulated trajectories. For instance, in case of the fixed leg, this is the part where the Best Estimate trajectories are utilized. Equation part $\boldsymbol{p_j(x,t)}$ is also gender dependent unlike the mortality rates used during the pricing process for the calculation of the life table aspects (**Equation 14 - Equation 16**) and commutation numbers (**Equation 17** and **Equation 18**). The reason behind is that males and females have significantly different mortality rates across ages. Although it is forbidden to consider this difference during pricing, it is relevant to be considered during the calculation of future obligations. Regarding the survival rate $(p_j(x,t))$ selection for individuals above age 100, the value used for person 'j' matches $p_j(100,t)$ in the age range 100 to 110, where „t” represents the year of reaching age 100. For instance, if person „j” purchases an immediate life annuity at age 65 in 2023, reaching age 100 occurs in 2058. Consequently, for the remaining observation period of 5 years, the survival rate utilized for person „j" is $p_j(100{,}2058)$. Additionally, for reference population cases aged 70-80 and 80-90 years old, the survival probability was considered as 0 for ages above 110. These assumptions were considered due to consistency reasons besides the lack of proper historical data in extreme age ranges. It is also essential to mention that the risk-free interest rates provided by the European Insurance and Occupational Pensions Authority (EIOPA) were used for the discount factor ($v(t)$) calculations, which were published in December 2023.

The results of the fixed leg and floating legs distribution achieved via **Equation 21** are presented in **Figure 26**, where the red line indicates the fixed leg. The return of the reinsurance contract in different floating leg scenarios was defined as the difference of the fixed leg and the floating legs. **Figure 27** presents the return distribution from the perspective of the reinsurer.

Figure 26, Distribution of the floating legs for reference population 60-70 ages (Hungary, without margin)



Source: Own calculations

Figure 27, Distribution of the expected return on the reinsurance contract for reference population 60-70 ages (Hungary, without margin)

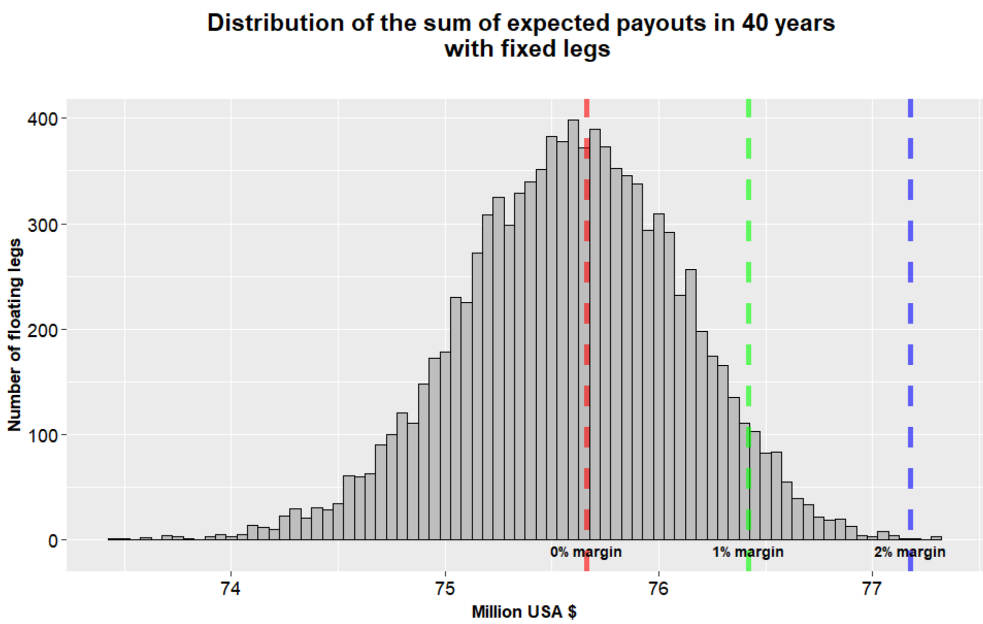
Source: Own calculations

Upon closer examination of **Figure 26** and **Figure 27** it becomes evident that the reinsurer, acting as the floating leg payer, realizes profit when the sum of expected payouts are below the amount of the fixed leg.

Besides the net scenario, two additional scenarios were inspected. In the first scenario the reinsurer added 1% extra margin, while in the second scenario 2% margin was added. The extra margins were applied on the fixed leg by shocking the underlying male and female Best Estimate mortality rate ($\boldsymbol{q_{x,t}}$) trajectories. As the fixed leg survivor rates ($\boldsymbol{p_j(x,t)}$) were determined through **Equation 14** based on the Best Estimate mortality rate trajectories ($\boldsymbol{q_{x,t}}$), the adjustments to the margins can be regarded as directly shocking the age and year specific survivor rates in **Equation 21**. Given that the reinsurer pays the floating leg and receives the fixed leg, it benefits from higher returns when mortality rates increase resulting in fewer obligations to fulfil. Conversely, if the mortality rates decrease then the reinsurer faces more obligations due to the fact that people live longer than anticipated. Therefore, during the margin calculations, the mortality rates were decreased by a shock percentage. It is important to note that the mortality rate was applied to the same extent to all the mortality rates across all ages.

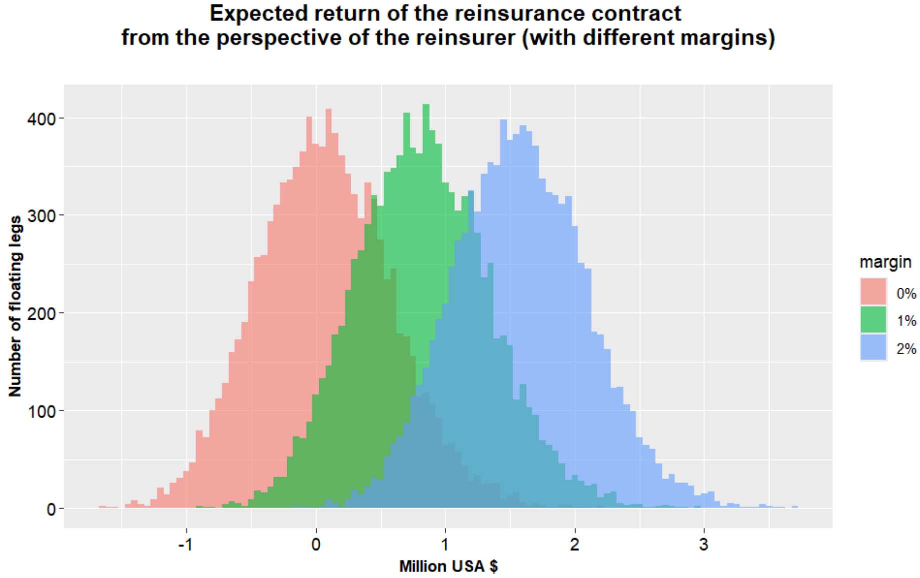
For 1% additional margin, a -10.03% shock was applied on the mortality rates. Similarly, 2% additional margin was achieved through -20.06% shock to mortality rates. This meant that both the original male and female Best Estimate trajectories changed to 89.97% in the former case and to 79.94% in the latter case compared to their defaults. When considering the change in the sum of expected payouts for the fixed leg, the net case (0% margin) yielded $75,663,572. With a 1% margin, the expected fixed leg payout increased to $76,420,208, and with a 2% margin, it further increased to $77,176,844. **Figure 28** and **Figure 29** illustrate the two additional scenarios with respect to the margins.

Figure 28, Distribution of the floating legs for reference population 60-70 ages (Hungary, with different margins)



Source: Own calculations

Figure 29, Distribution of the expected return on the reinsurance contract for reference population 60-70 ages (Hungary, with different margins)



Source: Own calculations

5 Results

Considering the hypotheses in question, the current chapter's aim is to provide answer to whether the age group of the population participating in the Longevity Reinsurance contract has an impact on the distribution of the return. Particularly focusing on the age groups of 60-70 as a benchmark compared to age groups 70-80 and 80-90. However, before presenting the direct results, **Table 9** emphasizes the features of the different age group cases. The exact age distributions of the observed populations are presented in **Appendix 5.**

Table 9, Gender and premium features of the simulated populations (Hungary)

	Male number	Female number	Sum of one-time premiums	Initial benefit payment in the first year
Population 60-70	489	511	$41,922,215	$5,400,855
Population 70-80	528	472	$41,292,907	$6,592,148
Population 80-90	507	493	$40,892,418	$9,678,596

Source: Own calculations

As **Table 9** illustrates, the sum of the one-time premiums was similar, consistent with the identical underlying exponential distributions. On the other hand, the sum of the yearly payouts differed significantly which is due to the change of annuity factor, presented in **Equation 19** and **Equation 20**. As for higher ages the value of the annuity factor decreases, and the payout value therefore increases inversely per individual. Consequently, the initial sum of payout obligations is higher for populations with older individuals. The detailed values per-age annuity factors are presented in **Appendix 6**. It is important to highlight, as annuity factors decrease at a higher rate than survival rates for older individuals, the sum of expected payout is also significantly higher for reference populations with older individuals. For instance, in case of the fixed legs (Best Estimate trajectories), the Population 60-70 had an expected payout amount of $75.663.572, the Population 70-80 had $81.840.167, and the Population 80-90 had $92,541,850. It may seem illogical if we consider that individuals in higher age groups have a greater probability of death, which would logically lead to a decrease in expected payouts over a 40-year period for elder reference populations. Furthermore, as presented in chapter 4.4, the survival probability was assumed to be 0 for ages above 110 which means that reference populations with 70-80 and 80-90 years olds have, on average, fewer years of payout obligations per individual compared to Population 60-70. Nevertheless, the relationship between the annuity factors and survival probabilities provides an adequate explanation for this outstanding effect. Specifically, the rate of decrease in annuity factors for older ages exceeds the rate of increase associated with mortality, consequently leading to greater expected payout amounts for elder reference populations.

For greater insight, the fixed leg values in cases of different margins are presented in **Table 10**, along with their percentile values relative to the distributions of the corresponding floating legs. The percentile percentage represents the proportion of floating legs which have sum of expected payout values less than the fixed leg value. Upon examining the percentile values, it becomes evident that even a relatively small percentage of margin pushes the fixed legs to the edge of the distributions. This can be regarded as a limitation of the current work, which may originate from the reference population sizes. As such, reference populations with more than 1,000 individuals may result in higher variability in individual expected payouts and therefore wider distributions.

Table 10, Fixed leg values in case of different margins for different reference populations (Hungary)

	Fixed leg – without margin		**Fixed leg – 1% margin**		**Fixed leg – 2% margin**	
	Value	Floating legs percentile	Value	Floating legs percentile	Value	Floating legs percentile
Population 60-70	$75,663,572	54.4197%	$76,420,208	94.9657%	$77,176,844	99.9650%
Population 70-80	$81,840,167	53.4877%	$82,658,569	85.8303%	$83,476,970	98.1137%
Population 80-90	$92,541,850	52.1017%	$93,467,269	78.8295%	$94,392,687	94.1343%

Source: Own calculations

Besides, **Table 10** also indicates that the floating leg percentile is above 50% in case of each population scenario if there is no additional margin applied. This discrepancy may be attributed to some form of Jensen's inequality, as the Best Estimate calculation of mortality rates for the fixed leg is performed before applying the main calculation formula, **Equation 21**. In contrast, the mean of the floating legs (50% percentile) is determined after estimating the 10.000 mortality rate trajectory simulation. However, this phenomenon is not examined in more detail and can therefore be regarded as a direction for future research. Additional information regarding the features of the floating legs distributions is presented in **Appendix 11**. Furthermore, detailed figures illustrating the relationship between the fixed and floating legs are provided in **Appendix 7** and **Appendix 8** for Population 70-80, and in **Appendix 9** and **Appendix 10** for Population 80-90.

As demonstrated at the end of chapter 4.4, the various margin scenarios were achieved through directs shocks to the mortality rates. Each margin scenario represents a constant change in mortality rates, thereby resulting a direct shift in the position of the fixed leg to ensure greater profitability for the reinsurer. The mortality shocks and the corresponding change in each level of the male and female Best Estimate mortality rates, compared to their default states are presented in **Table 11**, per reference population. The mortality shocks are defined as the necessary collective mortality changes to achieve the desired margin level.

Table 11, Margin shock to the male and female mortality rates per reference population (Hungary)

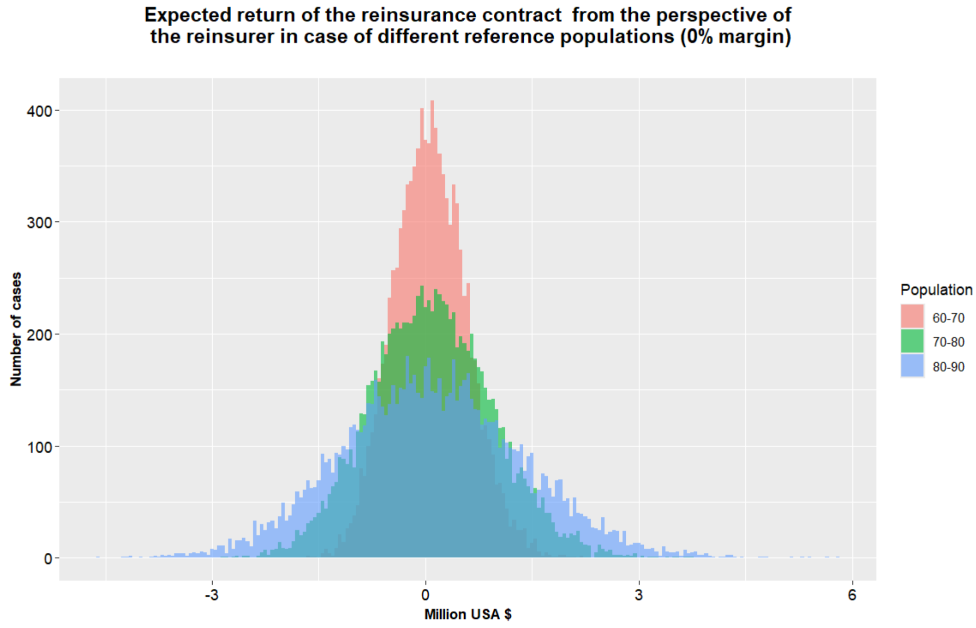
	Population 60-70		**Population 70-80**		**Population 80-90**	
Margin	1%	2%	1%	2%	1%	2%
Mortality shock	-10.03%	-20.06%	-5.38%	-10.77%	-3.01%	-6.02%
Level of mortality rates $(q_{x,t})$ after the shock	89.97%	79.94%	94.62%	89.23%	96.99%	93.98%

Source: Own calculations

Both **Table 10** and **Table 11** indicates that the distributions of the floating legs are affected by the underlying reference population. In case of **Table 10**, notable differences in the relationship between floating legs and fixed legs can be observed across different age-group cases, manifesting in the percentile position of the fixed legs. On the other hand, **Table 11** sheds light on the difference in the mortality shock required to achieve different margin levels, indicating different distribution shapes. While both tables foreshadowed that the return distribution of the Longevity Reinsurance contract is also dependent on the age composition of the reference population, Kolmogorov – Smirnov Tests (KS Test) were utilized to properly compare the distributions. The results of the KS Tests were identical, leading to the rejection of the null hypothesis of significant similarity between the observed return distributions at all usual confidence levels (1%, 5%, 10%). This conclusion remained the same across the different margin scenarios. Therefore, the research hypotheses H1, H2 and H3 could be accepted, which means that the return distribution of the Longevity Reinsurance contract is dependent on the age composition of the reinsured portfolio. The test statistics of the KS Tests are in **Appendix 12** for the net cases, in **Appendix 13** for the 1% margin cases and in **Appendix 14** for the 2% margin cases.

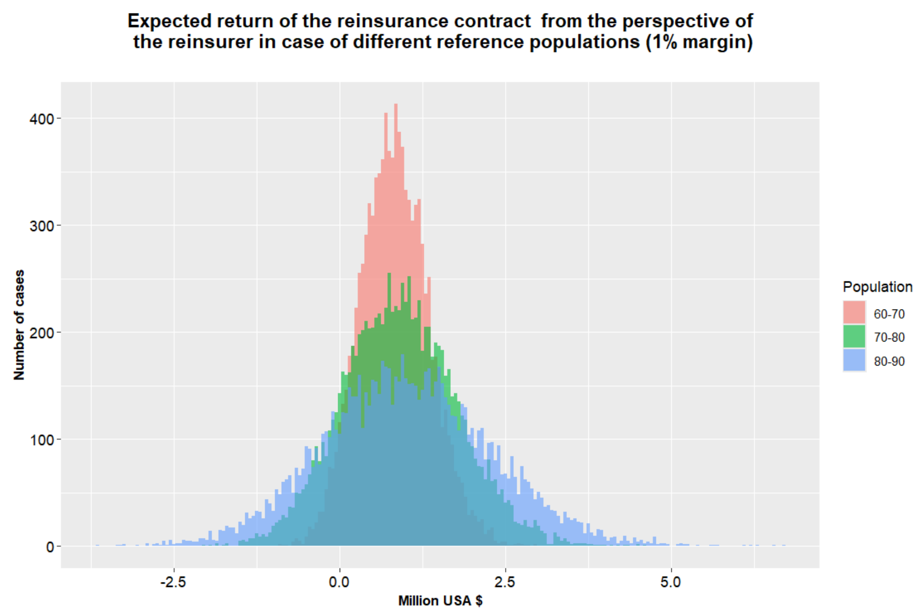
To provide visual support for the stated results, **Figure 30** presents the net return distributions in case of the different reference populations. Besides, **Figure 31** presents the 1% margin cases while **Figure 32** presents the 2% margin cases.

Figure 30, Distribution of the expected return in case of different reference populations (Hungary – 0% margin)



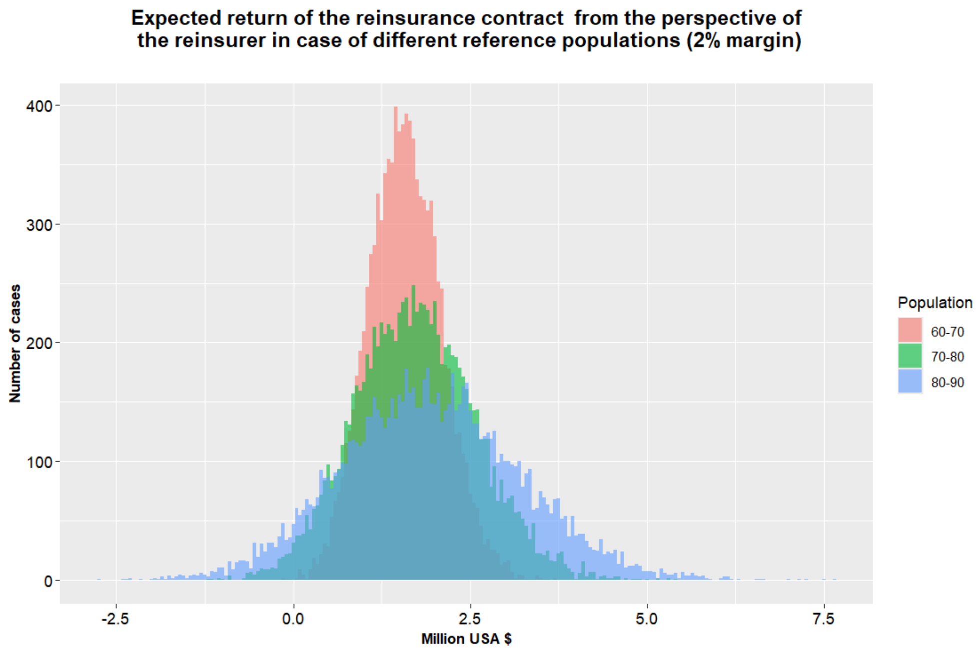
Source: Own calculations

Figure 31, Distribution of the expected return in case of different reference populations (Hungary – 1% margin)



Source: Own calculations

Figure 32, Distribution of the expected return in case of different reference populations (Hungary – 2% margin)



Source: Own calculations

Additionally, **Table 12-Table 14** present some of the relevant percentiles, derived form the return distributions illustrated in **Figure 30-Figure 32**. As a specific example, 1st (1%) percentile in **Table 12** indicates that the return of the reinsurer is above -1.0582 (million USA$) 99% of the time, assuming the underlying portfolio given into reinsurance consist of Hungarian individuals aged 60-70 years. For a portfolio of individuals aged 70-80 years, this value is -1.8166 (million USA$), while for a portfolio of individuals aged 80-90 years it is -2.7898 (million USD). The decrease in the percentile values for older portfolios is consistent with the finding that the standard deviation of the return increased for underlying reference portfolios consisting of older individuals. Therefore, there is greater risk associated with reinsuring portfolios that include older individuals. As shown in **Table 13** and **Table 14**, this effect remained consistent in cases when additional margins were applied to ensure the profitability of the reinsurer.

Table 12, Percentiles of the return distributions presented in Figure 30 (0% margin case) (million USA $)

	1%	5%	10%	50%
Population 60-70	-1.0582	-0.7585	-0.5841	0.0594
Population 70-80	-1.8166	-1.2911	-0.9914	0.0739
Population 80-90	-2.7898	-1.9517	-1.5055	0.0589

Source: Own calculations

Table 13, Percentiles of the return distributions presented in Figure 31 (1% margin case) (million USA $)

	1%	5%	10%	50%
Population 60-70	-0.3016	-0.0018	0.1726	0.8160
Population 70-80	-0.9982	-0.4727	-0.1730	0.8923
Population 80-90	-1.8644	-1.0263	-0.5800	0.9843

Source: Own calculations

Table 14, Percentiles of the return distributions presented in Figure 32 (2% margin case) (million USA $)

	1%	5%	10%	50%
Population 60-70	0.4550	0.7548	0.9292	1.5726
Population 70-80	-0.1798	0.3456	0.6454	1.7107
Population 80-90	-0.9390	-0.1009	0.3453	1.9097

Source: Own calculations

6 Conclusion

The aim of the present research was to provide a comprehensive overview of longevity risk management, in addition to gain better understanding of the return on Longevity Reinsurance contracts from the perspective of the reinsurer. In chapter 2.1 the theoretical background of mortality improvements was examined, followed by the longevity risk management methods in chapter 2.2. The overview of the theoretical background concluded with the presentation of mortality models in chapter 2.3, with particular emphasis on the Lee Carter model. After providing an overview of the theoretical background, chapter 3 outlined the research question and hypotheses. These were subsequently explored in chapter 4, where the used methodology was introduced, and in chapter 5, which presented the findings.

The main research question of the thesis explored the reinsurance contract's return when comparing populations of different age groups via simulation. Specifically, whether demographic factors, such as age groups, have significant impact on the return of Longevity Reinsurance contracts from the perspective of reinsurers. To investigate this question, one main and three sub-hypotheses were constructed. In summary, all research hypotheses were supported which meant that the distribution of the reinsurance contract was significantly dependent on the age composition of the reinsured portfolio. Especially, when comparing underlying portfolios consisting of individuals aged 60-70, 70-80, and 80-90, the return difference remained significant. Furthermore, as **Figure 30** illustrates, the return distributions were not only different, but the standard deviation of the return increased for underlying reference portfolios consisting of older individuals. In other words, there is greater risk associated with reinsuring portfolios that include older individuals. This effect remained consistent in cases when additional margins were applied to ensure the profitability of the reinsurer as **Figure 31** and **Figure 32** illustrate.

Considering the limitations of the current research it is important to mention the country selection. Because the focus was on Hungary, the inclusion of further countries could broaden the spectrum of the results. Therefore, this can be regarded as a future research direction. Similarly, by checking different forecast ranges and utilizing different initial assumptions, the reliability of the results can be greatly improved.

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8 Glossary

Annuity	Annuitás
Annuity factor	Annuitás faktor
Best estimate (BE)	Legjobb becslés
Central exposure to risk	Központi kitettség
Central rate of death	Központi halálozási ráta
Cost-of-capital	Tőkeköltség
Defined Benefit (DB) pension	Járadék alapú nyugdíjrendszer
Defined Contribution (DC) pension	Járulék alapú nyugdíjrendszer
Discount factor	Diszkont faktor
Discounted value of death	Halottak diszkontált száma
Discounted value of living	Élők diszkontált száma
Distribution	Eloszlás
Fixed leg	Csereügylet előre fixált lába
Floating leg	Csereügylet lebegő lába
Immediate life-time annuities	Azonnal induló életjáradék
Insurance industry	Biztosítási iparág
Life expectancy	várható élettartam
Longevity Bond	Hosszúélet kötvény

Longevity Insurance	Hosszúélet viszontbiztosítás
Longevity risk	Hosszúélet kockázat
Longevity Swaps	Hosszúélet csereügylet
Margin	Haszonrés (Marzs)
Mortality rates	Halálozási ráta
Number of deaths	Elhunytak száma
One-time premium	Egyszeri befizetés/egyszeri díj
Pension fund	Nyugdíj alap
Probability	Valószínűség
q-Forwards	Hosszúélet határidős csereügylet
Random Walk with Drift	Eltolásos véletlen bolyongás
Return	Megtérülés
Risk	Kockázat
Solvency Capital Requirement (SCR)	Szavatolótőke-szükséglet
Standard deviation	Szórás
Sum assured (SA)	Biztosítási összeg
Survival rate	Túlélési ráta
Technical interest rate	Technikai kamatláb
Underlying portfolio	Porfólió, amire az ügylet vonatkozik

9 Appendix

Appendix 1, R program script – Trajectories simulation

```
library(dplyr)
library(tidyr)
library(ggplot2)
library(demography)
library(MortalityLaws)
creation <- function(Country,CountryListPosition,
                     Ages,AgeMin,AgeMax,
                     Years,StartYear,EndYear,
                     CentralDeathRates,Exposures,
                     SimNum,SimYearNum,kt_adjust){

# Support variables
   start_time <- Sys.time()
     SupColNum <- SimNum*2 + 4
     SupColNum1 <- SimNum + 4
     SupColNum2 <- SupColNum1 + 1
     SupColNum4 <- SimNum + 2
     SupColNum5 <- SimNum + 5
     SupRowNum <- Ages * (SimYearNum + 1)
     SupRowNum1 <- Ages *SimYearNum
     SupRowStart <- Ages + 1
     SimYearNumPlus1 <- SimYearNum + 1
     SimEndYear <- EndYear+SimYearNum
     SupEndYear <- EndYear + 1

     # Initial data frame
     MainList <- list()
     MainList[["mxt_u"]] <- matrix(CentralDeathRates[[CountryListPosition]][,5], nrow = Ages, ncol = Years)
     MainList[["mxt_m"]] <- matrix(CentralDeathRates[[CountryListPosition]][,4], nrow = Ages, ncol = Years)
     MainList[["mxt_f"]] <- matrix(CentralDeathRates[[CountryListPosition]][,3], nrow = Ages, ncol = Years)
     MainList[["Ext_u"]] <- matrix(Exposures[[CountryListPosition]]$Total, nrow = Ages, ncol = Years)
     MainList[["Ext_m"]] <- matrix(Exposures[[CountryListPosition]]$Male, nrow = Ages, ncol = Years)
     MainList[["Ext_f"]] <- matrix(Exposures[[CountryListPosition]]$Female, nrow = Ages, ncol = Years)
     MainList[["support_u"]] <- demogdata(MainList$mxt_u, MainList$Ext_u, AgeMin:AgeMax,
                                 StartYear:EndYear, "mortality",Country, "Total")
     MainList[["support_m"]] <- demogdata(MainList$mxt_m, MainList$Ext_m, AgeMin:AgeMax,
```

```
                   StartYear:EndYear, "mortality",Country, "Male")
MainList[["support_f"]] <- demogdata(MainList$mxt_f, MainList$Ext_f, AgeMin:AgeMax,
                   StartYear:EndYear, "mortality",Country, "Female")

# Initial modelling
MainList[["LC_u"]] <- lca(data = MainList$support_u, max.age = AgeMax)
MainList[["LC_m"]] <- lca(data = MainList$support_m, max.age = AgeMax)
MainList[["LC_f"]] <- lca(data = MainList$support_f, max.age = AgeMax)

# Support Parameters
# Drift (c) estimation - Maxium Likelihod
MainList[["est_c_u"]] <- (MainList$LC_u$kt[Years]-MainList$LC_u$kt[1])/(Years)
MainList[["est_c_m"]] <- (MainList$LC_m$kt[Years]-MainList$LC_m$kt[1])/(Years)
MainList[["est_c_f"]] <- (MainList$LC_f$kt[Years]-MainList$LC_f$kt[1])/(Years)

# Standard error of deviation from linear change
standardError <- function(kt, drift){
  support <- as.numeric()
  sd <- as.numeric()
  for(k in 2:length(kt)){
    support[k-1] <- (kt[k]-kt[k-1]-drift)^2}
  sd <- sqrt(sum(support)/length(kt))
  return(sd)}
MainList[["est_se_u"]] <- standardError(MainList$LC_u$kt, MainList$est_c_u)
MainList[["est_se_m"]] <- standardError(MainList$LC_m$kt, MainList$est_c_m)
MainList[["est_se_f"]] <- standardError(MainList$LC_f$kt, MainList$est_c_f)
MainList[["est_c_sd_u"]] <- MainList$est_se_u/sqrt(Years)
MainList[["est_c_sd_m"]] <- MainList$est_se_m/sqrt(Years)
MainList[["est_c_sd_f"]] <- MainList$est_se_f/sqrt(Years)

# Simulation
kt_forecast <- function(sim_num,year_num,kT, est_c, est_c_sd, est_se){
  row_num <- year_num +1
  output <- data.frame(matrix(ncol=sim_num, nrow = row_num))
  for(world in 1:sim_num){
    dist1 <- rnorm(year_num, mean=0, sd =1)
    dist2 <- rnorm(year_num, mean=0, sd =1)
    output[1,world] <- kT
    support <- dist2[1]
    for(year in 2:row_num){
```

```
    output[year,world] <- output[1,world]+(est_c-est_c_sd*dist1[year-1])*(year-1)+est_se*support
    support <- support+dist2[year]}}
  return(output)}
MainList[["kT_u"]] <- MainList$LC_u$kt[Years]
MainList[["kT_m"]] <- MainList$LC_m$kt[Years]
MainList[["kT_f"]] <- MainList$LC_f$kt[Years]
MainList[["kt_future_u"]] <- kt_forecast(SimNum,SimYearNum,MainList$kT_u,
                                          MainList$est_c_u,MainList$est_c_sd_u,
                                          MainList$est_se_u)
MainList[["kt_future_m"]] <- kt_forecast(SimNum,SimYearNum,MainList$kT_m,
                                          MainList$est_c_m,MainList$est_c_sd_m,
                                          MainList$est_se_m)
MainList[["kt_future_f"]] <- kt_forecast(SimNum,SimYearNum,MainList$kT_f,
                                          MainList$est_c_f,MainList$est_c_sd_f,
                                          MainList$est_se_f)
MainList[["kt_future_u"]]$year <- EndYear:SimEndYear
MainList[["kt_future_m"]]$year <- EndYear:SimEndYear
MainList[["kt_future_f"]]$year <- EndYear:SimEndYear

# Creating data frame
MainList[["mx_forecast_u"]]<- data.frame(matrix(nrow = SupRowNum, ncol = SupColNum))
MainList[["mx_forecast_m"]]<- data.frame(matrix(nrow = SupRowNum, ncol = SupColNum))
MainList[["mx_forecast_f"]]<- data.frame(matrix(nrow = SupRowNum, ncol = SupColNum))
MainList[["mx_forecast_u"]] <- MainList[["mx_forecast_u"]] %>%
  rename_at(vars(1:4), ~ c("Year", "Age", "ax", "bx"))
MainList[["mx_forecast_m"]] <- MainList[["mx_forecast_m"]] %>%
  rename_at(vars(1:4), ~ c("Year", "Age", "ax", "bx"))
MainList[["mx_forecast_f"]] <- MainList[["mx_forecast_f"]] %>%
  rename_at(vars(1:4), ~ c("Year", "Age", "ax", "bx"))
MainList[["mx_forecast_u"]] <- MainList[["mx_forecast_u"]] %>%
  rename_at(vars(5:SupColNum1), ~ paste("kt_forecast", 1:SimNum, sep = "_"))
MainList[["mx_forecast_m"]] <- MainList[["mx_forecast_m"]] %>%
  rename_at(vars(5:SupColNum1), ~ paste("kt_forecast", 1:SimNum, sep = "_"))
MainList[["mx_forecast_f"]] <- MainList[["mx_forecast_f"]] %>%
  rename_at(vars(5:SupColNum1), ~ paste("kt_forecast", 1:SimNum, sep = "_"))
MainList[["mx_forecast_u"]] <- MainList[["mx_forecast_u"]] %>%
  rename_at(vars(SupColNum2:SupColNum ), ~ paste("mx_forecast", 1:SimNum, sep = "_"))
MainList[["mx_forecast_m"]] <- MainList[["mx_forecast_m"]] %>%
  rename_at(vars(SupColNum2:SupColNum ), ~ paste("mx_forecast", 1:SimNum, sep = "_"))
MainList[["mx_forecast_f"]] <- MainList[["mx_forecast_f"]] %>%
```

```
  rename_at(vars(SupColNum2:SupColNum ), ~ paste("mx_forecast", 1:SimNum, sep = "_"))

# year, age, ax, bx, kt
MainList$mx_forecast_u$Year <- rep(EndYear:SimEndYear, each = Ages)
MainList$mx_forecast_u$Age <- rep(AgeMin:AgeMax, times = SimYearNumPlus1)
MainList$mx_forecast_u$ax <- rep(MainList$LC_u$ax, times = SimYearNumPlus1)
MainList$mx_forecast_u$bx <- rep(MainList$LC_u$bx, times = SimYearNumPlus1)
for(col in 5:SupColNum1)
{MainList$mx_forecast_u[,col] <- rep(MainList$kt_future_u[,col-4], each = Ages)}

MainList$mx_forecast_m$Year <- rep(EndYear:SimEndYear, each = Ages)
MainList$mx_forecast_m$Age <- rep(AgeMin:AgeMax, times = SimYearNumPlus1)
MainList$mx_forecast_m$ax <- rep(MainList$LC_m$ax, times = SimYearNumPlus1)
MainList$mx_forecast_m$bx <- rep(MainList$LC_m$bx, times = SimYearNumPlus1)
for(col in 5:SupColNum1)
{MainList$mx_forecast_m[,col] <- rep(MainList$kt_future_m[,col-4], each = Ages)}

MainList$mx_forecast_f$Year <- rep(EndYear:SimEndYear, each = Ages)
MainList$mx_forecast_f$Age <- rep(AgeMin:AgeMax, times = SimYearNumPlus1)
MainList$mx_forecast_f$ax <- rep(MainList$LC_f$ax, times = SimYearNumPlus1)
MainList$mx_forecast_f$bx <- rep(MainList$LC_f$bx, times = SimYearNumPlus1)
for(col in 5:SupColNum1)
{MainList$mx_forecast_f[,col] <- rep(MainList$kt_future_f[,col-4], each = Ages)}

# mx calculation
for(row in SupRowStart:SupRowNum){
  for(col in SupColNum2:SupColNum){
    MainList$mx_forecast_u[row,col] <- exp(MainList$mx_forecast_u$ax[row]+
                                    MainList$mx_forecast_u$bx[row]*
                                    MainList$mx_forecast_u[row,col-SimNum])
    MainList$mx_forecast_m[row,col] <- exp(MainList$mx_forecast_m$ax[row]+
                                    MainList$mx_forecast_m$bx[row]*
                                    MainList$mx_forecast_m[row,col-SimNum])
    MainList$mx_forecast_f[row,col] <- exp(MainList$mx_forecast_f$ax[row]+
                                    MainList$mx_forecast_f$bx[row]*
                                    MainList$mx_forecast_f[row,col-SimNum])}}
MainList$mx_forecast_u <- MainList$mx_forecast_u[c(SupRowStart:SupRowNum),]
MainList$mx_forecast_m <- MainList$mx_forecast_m[c(SupRowStart:SupRowNum),]
MainList$mx_forecast_f <- MainList$mx_forecast_f[c(SupRowStart:SupRowNum),]
```

```
# qx ML estimation
MainList[["qx_forecast_u"]]<- data.frame(matrix(nrow = SupRowNum1, ncol = SupColNum4))
MainList[["qx_forecast_m"]]<- data.frame(matrix(nrow = SupRowNum1, ncol = SupColNum4))
MainList[["qx_forecast_f"]]<- data.frame(matrix(nrow = SupRowNum1, ncol = SupColNum4))
MainList[["qx_forecast_u"]] <- MainList[["qx_forecast_u"]] %>%
  rename_at(vars(1:2), ~ c("Year", "Age"))
MainList[["qx_forecast_m"]] <- MainList[["qx_forecast_m"]] %>%
  rename_at(vars(1:2), ~ c("Year", "Age"))
MainList[["qx_forecast_f"]] <- MainList[["qx_forecast_f"]] %>%
  rename_at(vars(1:2), ~ c("Year", "Age"))
MainList[["qx_forecast_u"]] <- MainList[["qx_forecast_u"]] %>%
  rename_at(vars(3:SupColNum4), ~ paste("qx_forecast", 1:SimNum, sep = "_"))
MainList[["qx_forecast_m"]] <- MainList[["qx_forecast_m"]] %>%
  rename_at(vars(3:SupColNum4), ~ paste("qx_forecast", 1:SimNum, sep = "_"))
MainList[["qx_forecast_f"]] <- MainList[["qx_forecast_f"]] %>%
  rename_at(vars(3:SupColNum4), ~ paste("qx_forecast", 1:SimNum, sep = "_"))
MainList$qx_forecast_u$Year <- rep(SupEndYear:SimEndYear, each = Ages)
MainList$qx_forecast_m$Year <- rep(SupEndYear:SimEndYear, each = Ages)
MainList$qx_forecast_f$Year <- rep(SupEndYear:SimEndYear, each = Ages)
MainList$qx_forecast_u$Age <- rep(AgeMin:AgeMax, times = SimYearNum)
MainList$qx_forecast_m$Age <- rep(AgeMin:AgeMax, times = SimYearNum)
MainList$qx_forecast_f$Age <- rep(AgeMin:AgeMax, times = SimYearNum)

# qx Calculation
for(row in 1:SupRowNum1){
  for(col in SupColNum5:SupColNum ){
    MainList$qx_forecast_u[row,col-SupColNum4] <- 1-exp(-MainList$mx_forecast_u[row,col])
    MainList$qx_forecast_m[row,col-SupColNum4] <- 1-exp(-MainList$mx_forecast_m[row,col])
    MainList$qx_forecast_f[row,col-SupColNum4] <- 1-exp(-MainList$mx_forecast_f[row,col])}}

return(MainList)}
```

Example – Hungary

```
HU <- creation("HU",1,101,0,100,55,1966,2020,deathRates,exposures,1000,43,FALSE)
```

Appendix 2, R program script – Best Estimate trajectories simulation (Hungary)

```
HU_BE <- list()
# Forecast - Best Estimate (BE)
  HU_BE[["BE_future_u" ]] <- forecast(HU$LC_u, h =43 )
  HU_BE[["BE_future_m" ]] <- forecast(HU$LC_m, h =43 )
  HU_BE[["BE_future_f" ]] <- forecast(HU$LC_f, h =43 )
# Forecast of the rates
  HU_BE[["BE_mxt_u" ]] <-HU_BE$BE_future_u$rate$Total
  HU_BE[["BE_mxt_m" ]] <-HU_BE$BE_future_m$rate$Male
  HU_BE[["BE_mxt_f" ]] <-HU_BE$BE_future_f$rate$Female
  HU_BE[["BE_qx_u" ]] <- 1-exp(-HU_BE$BE_mxt_u)
  HU_BE[["BE_qx_m" ]] <- 1-exp(-HU_BE$BE_mxt_m)
  HU_BE[["BE_qx_f" ]] <- 1-exp(-HU_BE$BE_mxt_f)

# Organising qx values to a data frame
  HU_BE[["BE_qx_df_u"]] <- data.frame(matrix(nrow=(101*43),ncol = 3))
  HU_BE[["BE_qx_df_m"]] <- data.frame(matrix(nrow=(101*43),ncol = 3))
  HU_BE[["BE_qx_df_f"]] <- data.frame(matrix(nrow=(101*43),ncol = 3))
  names(HU_BE$BE_qx_df_u) <- c("Years","Age","qx_BE")
  names(HU_BE$BE_qx_df_m) <- c("Years","Age","qx_BE")
  names(HU_BE$BE_qx_df_f) <- c("Years","Age","qx_BE")

  HU_BE$BE_qx_df_u$Years <- HU$qx_forecast_u$Year
  HU_BE$BE_qx_df_u$Age <- HU$qx_forecast_u$Age
  HU_BE$BE_qx_df_u$qx_BE <- c(HU_BE$BE_qx_u)

  HU_BE$BE_qx_df_m$Years <- HU$qx_forecast_m$Year
  HU_BE$BE_qx_df_m$Age <- HU$qx_forecast_m$Age
  HU_BE$BE_qx_df_m$qx_BE <- c(HU_BE$BE_qx_m)

  HU_BE$BE_qx_df_f$Years <- HU$qx_forecast_f$Year
  HU_BE$BE_qx_df_f$Age <- HU$qx_forecast_f$Age
  HU_BE$BE_qx_df_f$qx_BE <- c(HU_BE$BE_qx_f)
```

Appendix 3, Age distribution of the simulated population of 60-70 years olds

Age	Count	Proportion
60	42	4.2%
61	103	10.3%
62	92	9.2%
63	108	10.8%
64	98	9.8%
65	91	9.1%
66	124	12.4%
67	103	10.3%
68	96	9.6%
69	103	10.3%
70	40	4.0%

Source: Own calculations

Appendix 4, Calculation of Hungarian median retirement savings (2022)

Country	Unit	Average annual wage	Ratio	Retirement saving (λ)
USA	US Dollar, 2022	$ 77,463	100%	$87,000
Hungary	US Dollar, 2022	$ 28,475	37%	$31,981

Source: Own calculations

Appendix 5, The age distribution of observed populations (Hungary)

Population 60-70			Population 70-80			Population 80-90		
Age	**Person count**	**Proportion**	**Age**	**Person count**	**Proportion**	**Age**	**Person count**	**Proportion**
60	42	4.2%	**70**	46	4.6%	**80**	47	4.7%
61	103	10.3%	**71**	115	11.5%	**81**	96	9.6%
62	92	9.2%	**72**	99	9.9%	**82**	95	9.5%
63	108	10.8%	**73**	89	8.9%	**83**	95	9.5%
64	98	9.8%	**74**	102	10.2%	**84**	92	9.2%
65	91	9.1%	**75**	116	11.6%	**85**	95	9.5%
66	124	12.4%	**76**	98	9.8%	**86**	114	11.4%
67	103	10.3%	**77**	84	8.4%	**87**	112	11.2%
68	96	9.6%	**78**	105	10.5%	**88**	92	9.2%
69	103	10.3%	**79**	93	9.3%	**89**	112	11.2%
70	40	4.0%	**80**	53	5.3%	**90**	50	5.0%

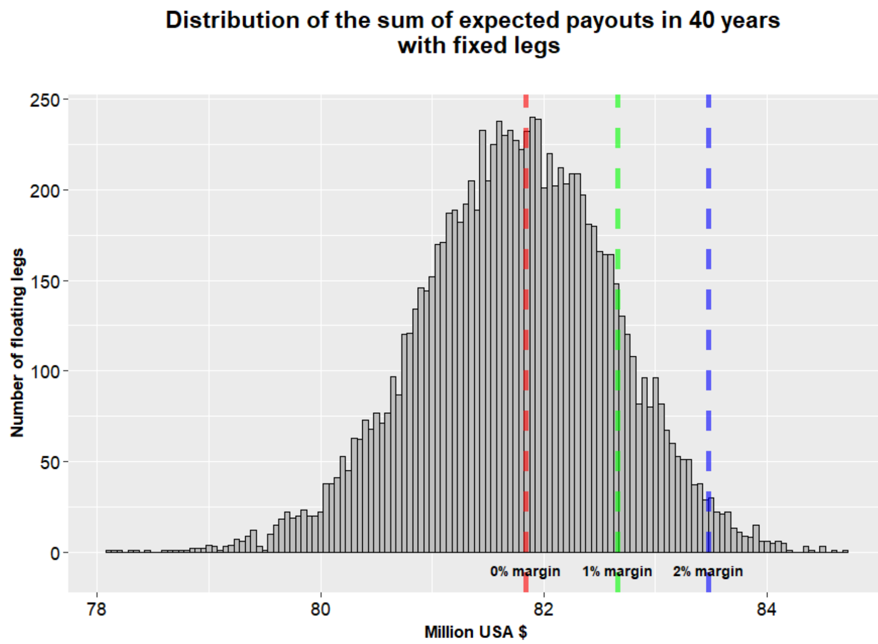
Source: Own calculations

Appendix 6, Pricing annuity factor (immediate life annuities) values for different ages

Population 60-70		Population 70-80		Population 80-90	
Entry year	**Pricing annuity**	**Entry year**	**Pricing annuity**	**Entry year**	**Pricing annuity**
60	8.31885	70	7.14769	80	5.30342
61	8.22505	71	6.99305	81	5.09959
62	8.12742	72	6.82976	82	4.89750
63	8.02534	73	6.66082	83	4.69477
64	7.91918	74	6.48326	84	4.49838
65	7.80923	75	6.29777	85	4.29896
66	7.69290	76	6.10795	86	4.10951
67	7.56774	77	5.91206	87	3.92503
68	7.43549	78	5.71228	88	3.74641
69	7.29564	79	5.50985	89	3.57070
70	7.14769	80	5.30342	90	3.39587

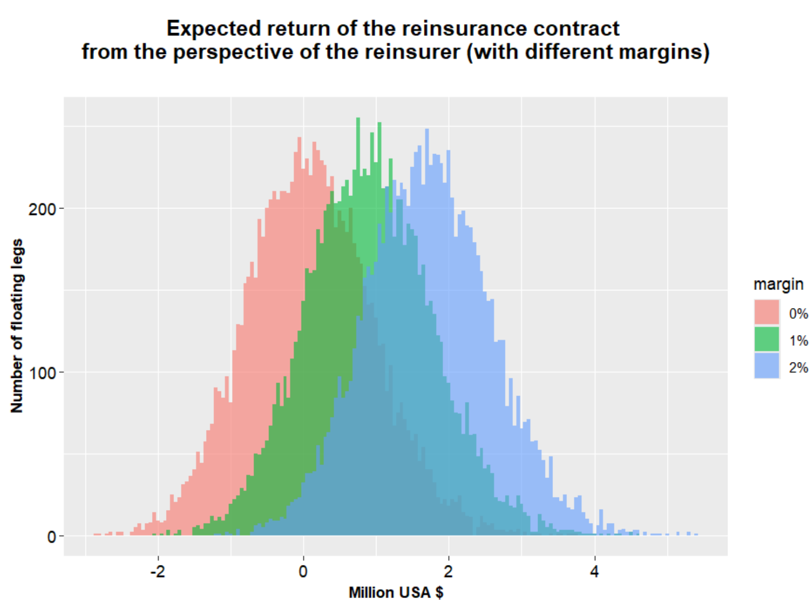
Source: Own calculations

Appendix 7, Distribution of the floating legs for reference population 70-80 ages (Hungary, with different margins)



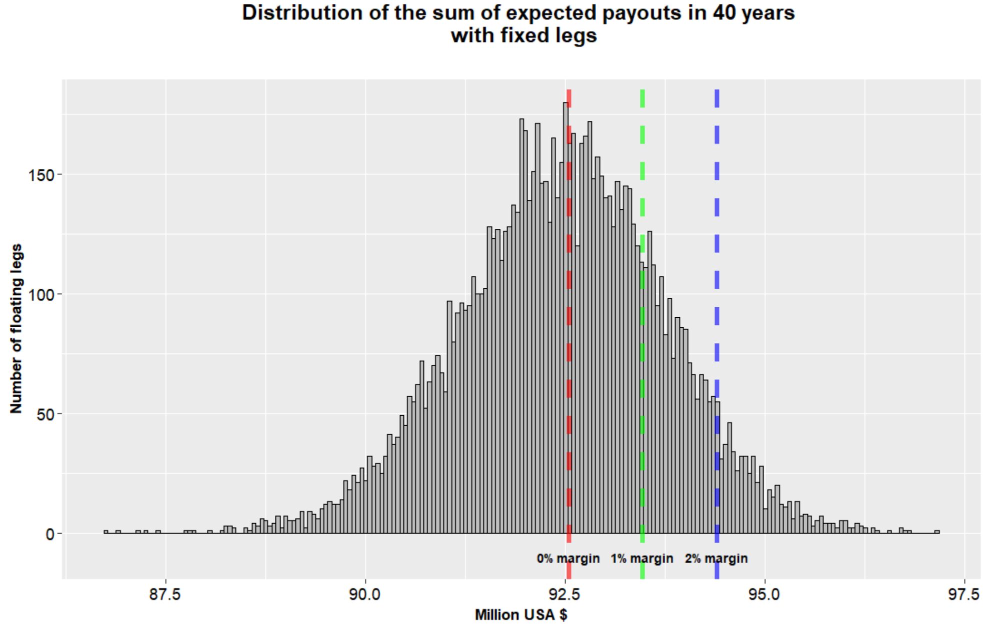
Source: Own calculations

Appendix 8 , Distribution of the expected return on the reinsurance contract for reference population 70-80 ages (Hungary, with different margins)



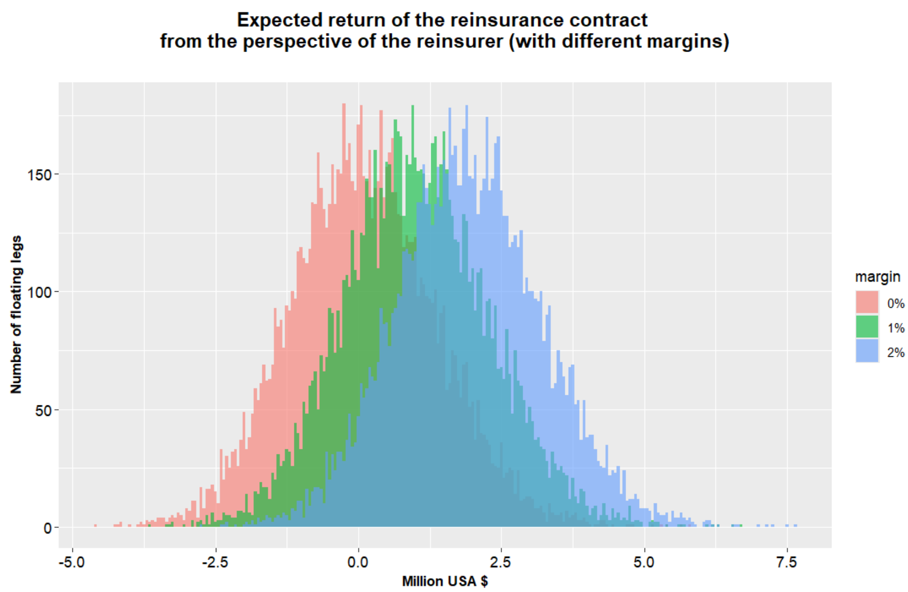
Source: Own calculations

Appendix 9, Distribution of the floating legs for reference population 80-90 ages (Hungary, with different margins)



Source: Own calculations

Appendix 10 , Distribution of the expected return on the reinsurance contract for reference population 80-90 ages (Hungary, with different margins)



Source: Own calculations

Appendix 11, Features of the floating leg distributions per reference population (Hungary)

	Minimum	Q1 quartile	Q2 quartile (Mean)	Q3 quartile	Maximum
Population 60-70	$73,474,235	$75,245,575	$75,604,210	$75,953,562	$77,323,363
Population 70-80	$78,459,868	$81,127,033	$81,737,470	$82,346,763	$84,090,898
Population 80-90	$87,745,333	$91,596,345	$92,470,983	$93,340,101	$96,033,335

Source: Own calculations

Appendix 12, Test statistics of the Kolmogorov-Smirnov tests (Different age groups case – without margin)

Without margin (0%)

	Population 60-70	Population 70-80	Population 80-90
Population 60-70			
Population 70-80	**D = 0.1273** **p-value < 0.0000000**		
Population 80-90	**D = 0.2084** **p-value < 0.0000000**	**D = 0.098** **p-value < 0.0000000**	

Source: Own calculations

Appendix 13, Test statistics of the Kolmogorov-Smirnov tests
(Different age groups case – 1% margin)

Without margin (1%)

	Population 60-70	Population 70-80	Population 80-90
Population 60-70			
Population 70-80	D = 0.1504 p-value < 0.0000000		
Population 80-90	D = 0.2483 p-value < 0.0000000	D = 0.1164 p-value < 0.0000000	

Source: Own calculations

Appendix 14, Test statistics of the Kolmogorov-Smirnov tests
(Different age groups case – 2% margin)

Without margin (2%)

	Population 60-70	Population 70-80	Population 80-90
Population 60-70			
Population 70-80	D = 0.1752 p-value < 0.0000000		
Population 80-90	D = 0.2998 p-value < 0.0000000	D = 0.1423 p-value < 0.0000000	

Source: Own calculations